

UCLA

**Computer
Science**



From Probabilistic Circuits to Probabilistic Programs and Back

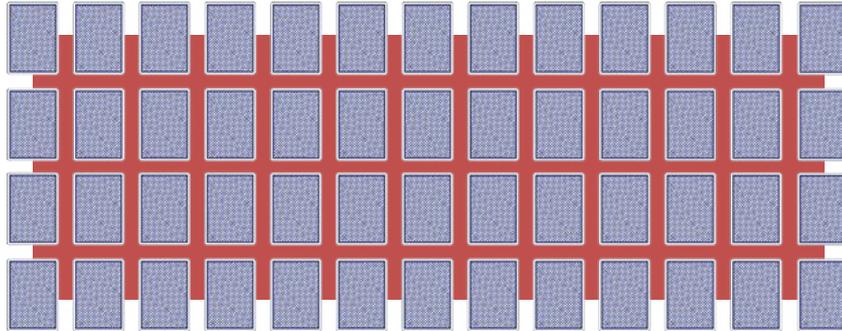
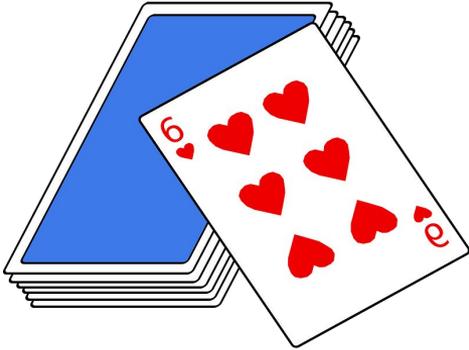
Guy Van den Broeck

ICAART - Feb 6, 2021

Trying to be provocative

Probabilistic graphical models is how we do probabilistic AI!

Graphical models of variable-level (in)dependence are a broken abstraction.

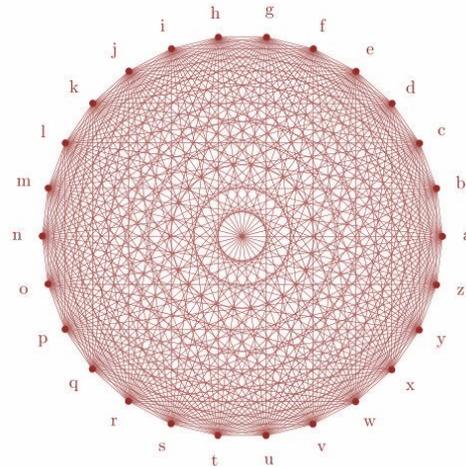


Trying to be provocative

Probabilistic graphical models is how we do probabilistic AI!

Graphical models of variable-level (in)dependence are a broken abstraction.

3.14 $\text{Smokes}(x) \wedge \text{Friends}(x,y)$
 $\Rightarrow \text{Smokes}(y)$



Trying to be provocative

Probabilistic graphical models is how we do probabilistic AI!

Graphical models of variable-level (in)dependence are a broken abstraction.

Bean Machine

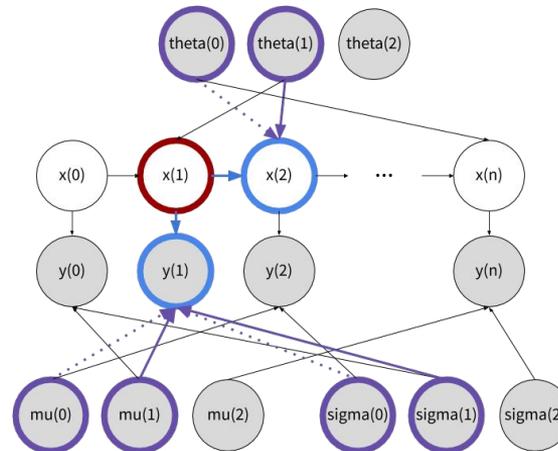
$$\mu_k \sim \text{Normal}(\alpha, \beta)$$

$$\sigma_k \sim \text{Gamma}(\nu, \rho)$$

$$\theta_k \sim \text{Dirichlet}(\kappa)$$

$$x_i \sim \begin{cases} \text{Categorical}(init) & \text{if } i = 0 \\ \text{Categorical}(\theta_{x_{i-1}}) & \text{if } i > 0 \end{cases}$$

$$y_i \sim \text{Normal}(\mu_{x_i}, \sigma_{x_i})$$



Computational Abstractions

Let us think of probability distributions as objects that are computed.

Abstraction = Structure of Computation

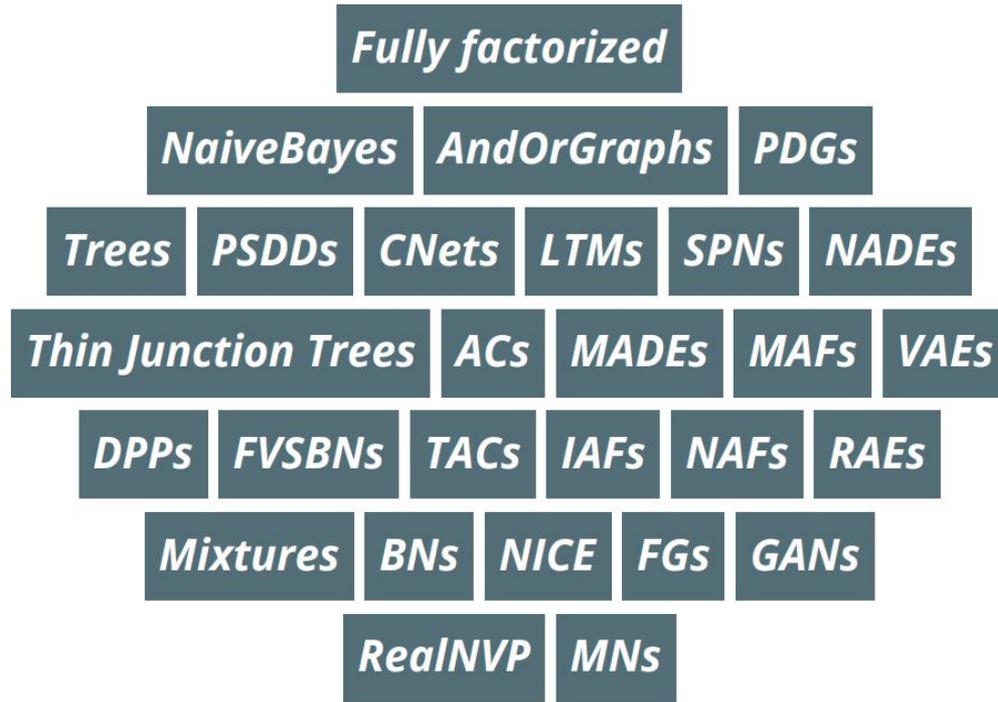
Two examples:

1. Probabilistic Circuits
2. Probabilistic Programs

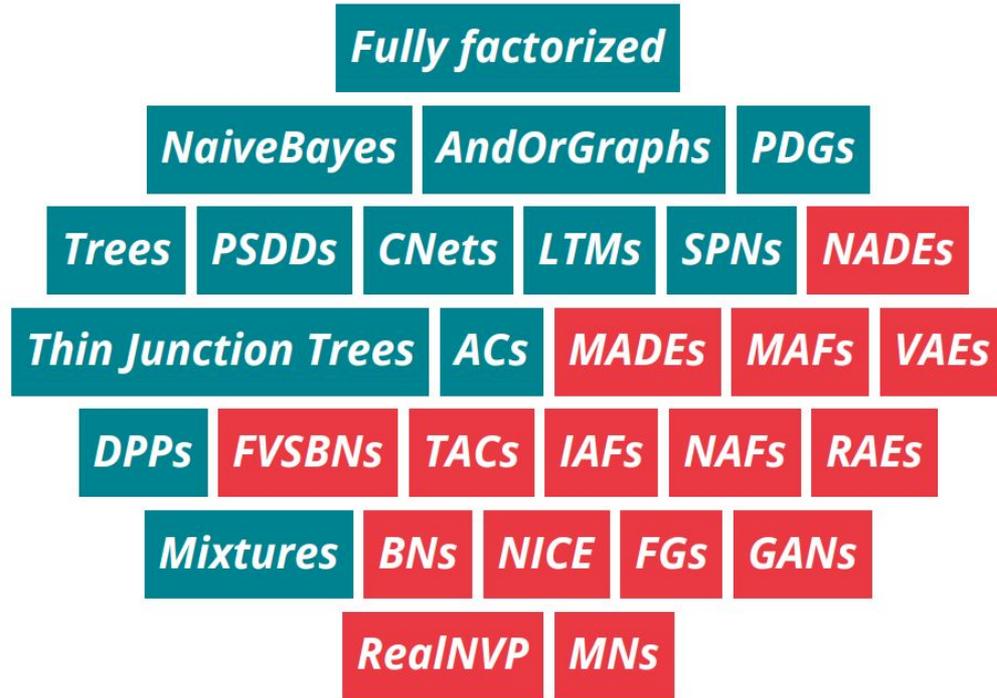


Probabilistic Circuits



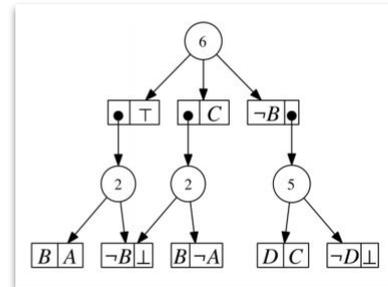
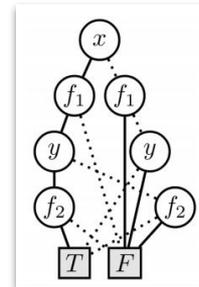
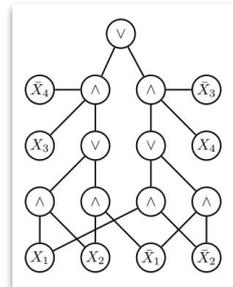
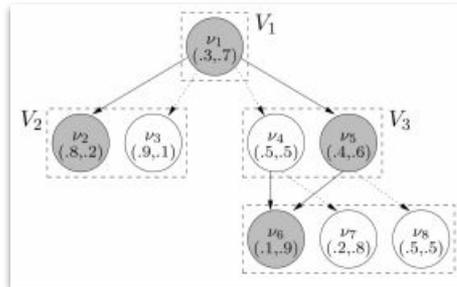
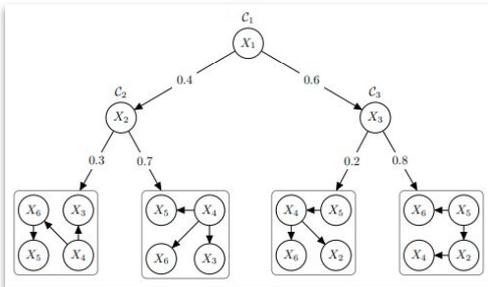
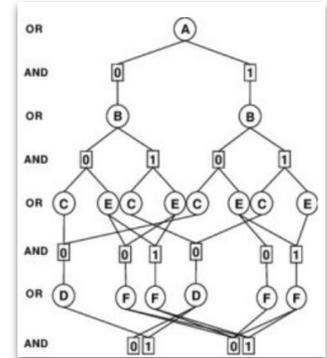
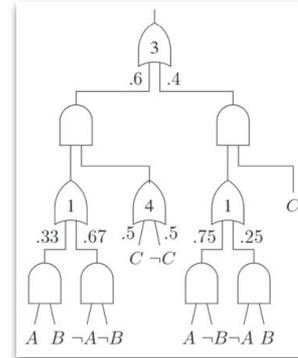
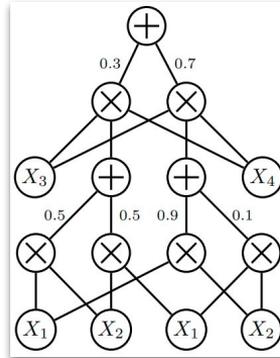
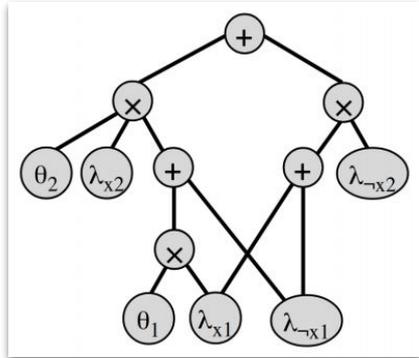
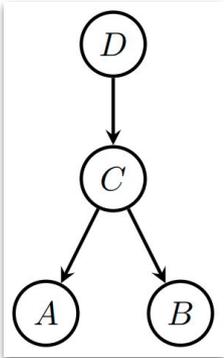


The Alphabet Soup of probabilistic models

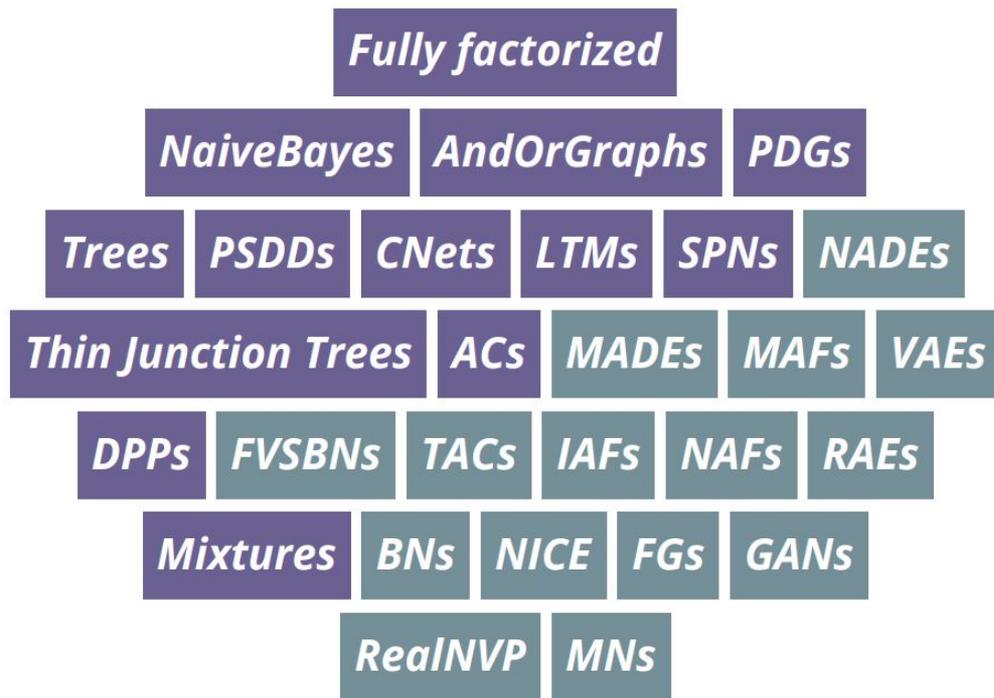


Intractable and ***tractable*** models

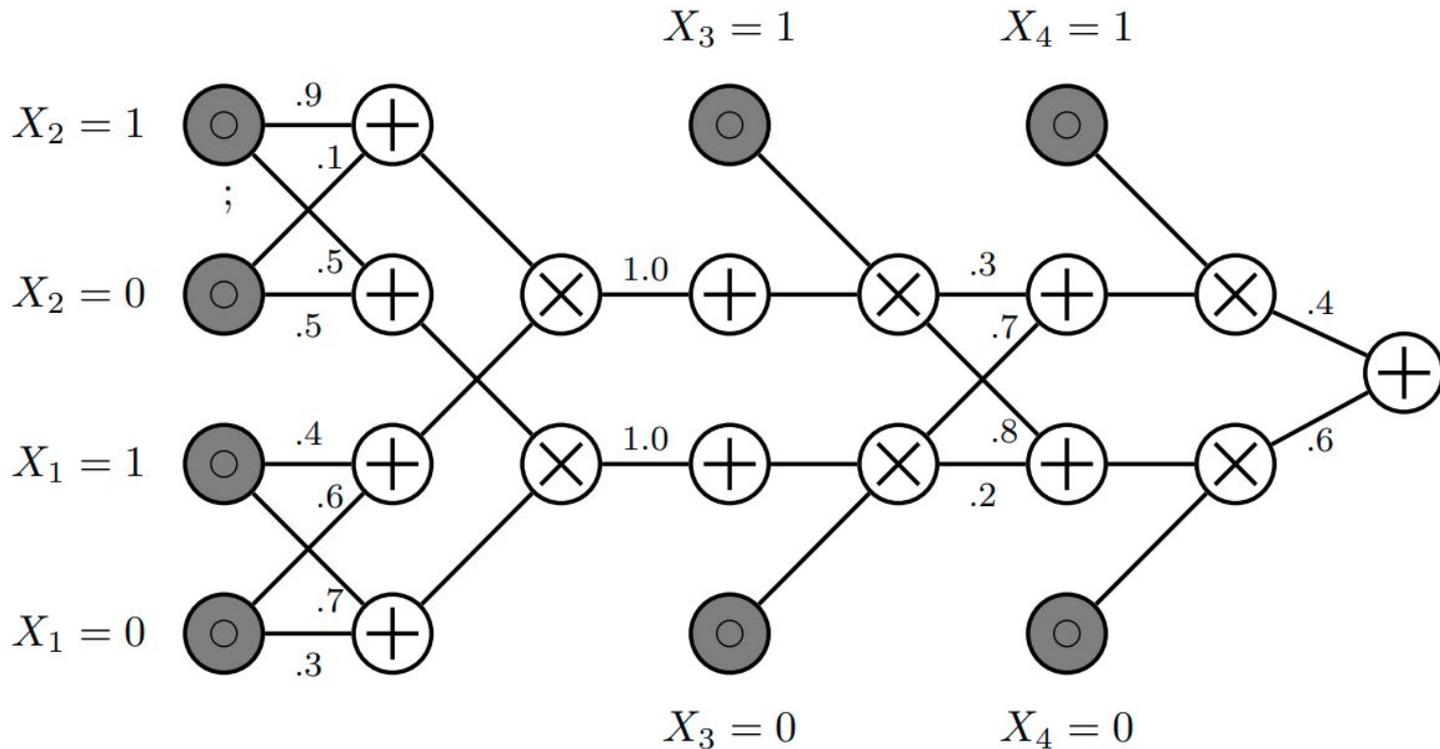
Tractable Probabilistic Models



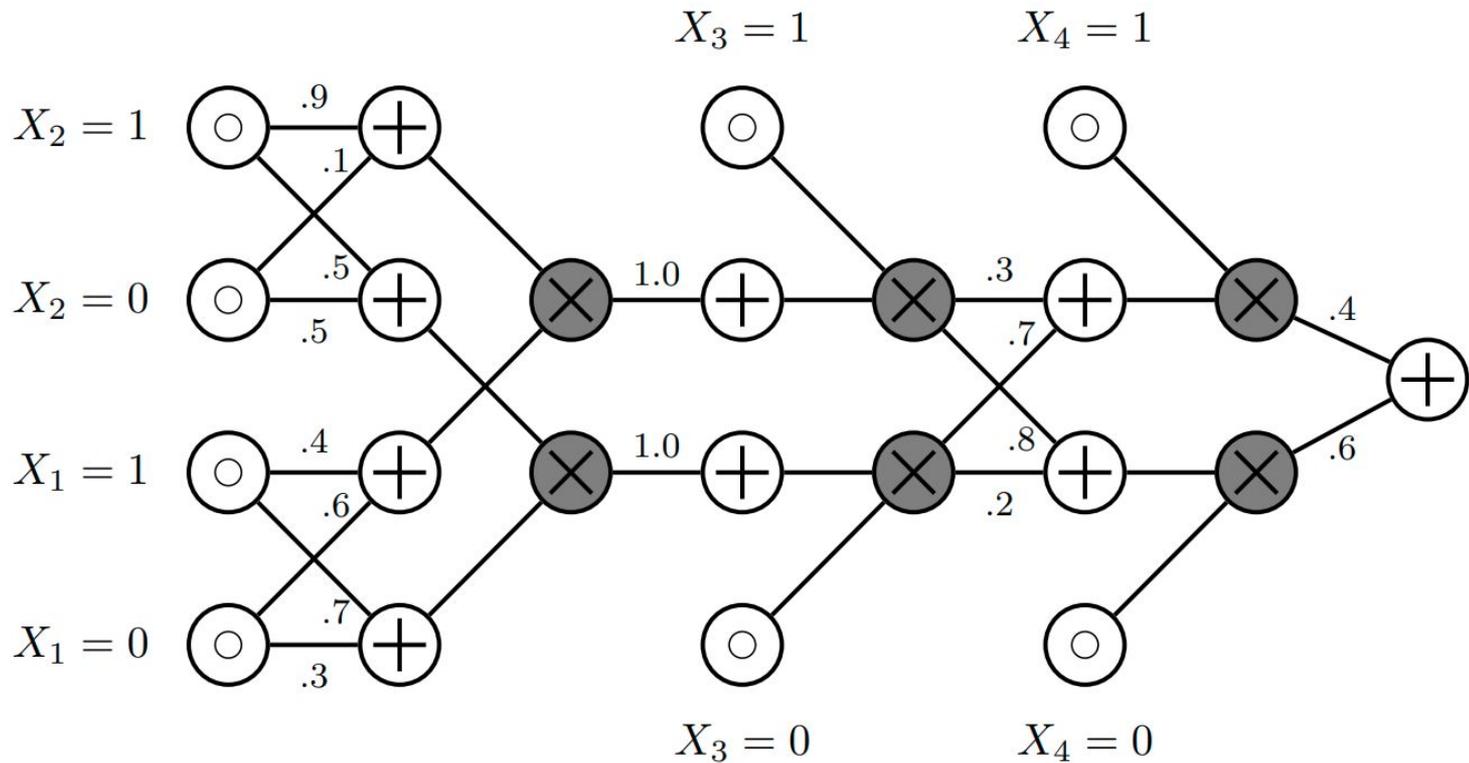
"Every talk needs a joke and a literature overview slide, not necessarily distinct"
 - after Ron Graham



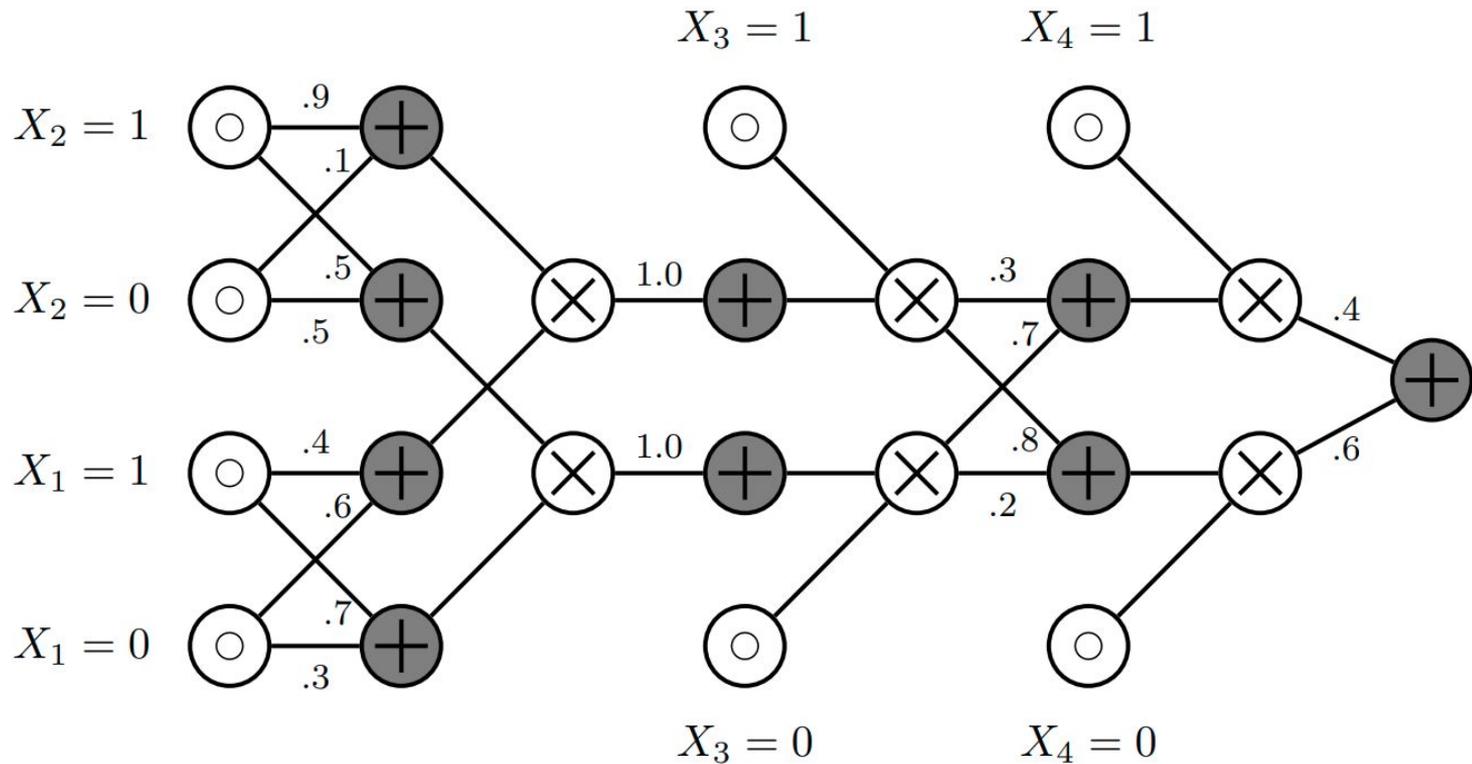
***a unifying framework* for tractable models**



Input nodes c are tractable (simple) distributions, e.g., univariate gaussian or indicator $p_c(X=1) = [X=1]$



Product nodes are factorizations $\prod_{c \in \text{in}(n)} p_c(\mathbf{x})$



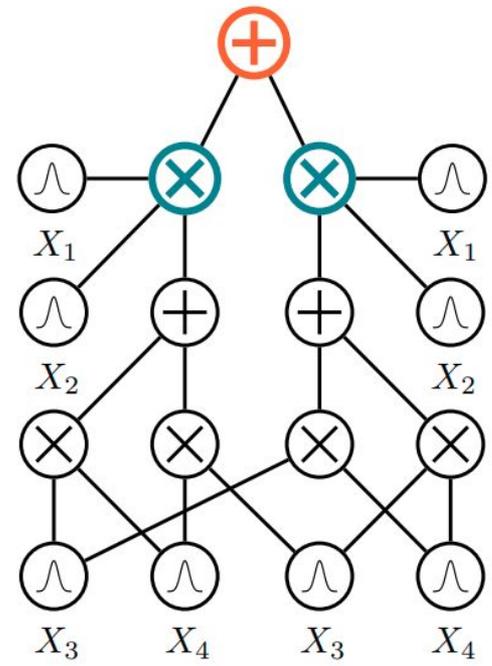
Sum nodes are mixture models $\sum_{c \in \text{in}(n)} \theta_{n,c} p_c(\mathbf{x})$

Smoothness + decomposability = tractable MAR

If $p(\mathbf{x}) = \sum_i w_i p_i(\mathbf{x})$, (**smoothness**):

$$\int p(\mathbf{x}) d\mathbf{x} = \int \sum_i w_i p_i(\mathbf{x}) d\mathbf{x} = \sum_i w_i \int p_i(\mathbf{x}) d\mathbf{x}$$

\Rightarrow integrals are "pushed down" to children

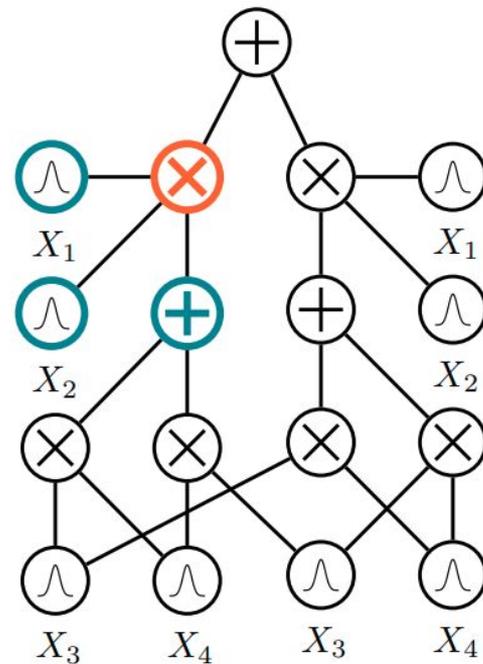


Smoothness + decomposability = tractable MAR

If $p(\mathbf{x}, \mathbf{y}, \mathbf{z}) = p(\mathbf{x})p(\mathbf{y})p(\mathbf{z})$, (**decomposability**):

$$\begin{aligned} & \int \int \int p(\mathbf{x}, \mathbf{y}, \mathbf{z}) dx dy dz = \\ &= \int \int \int p(\mathbf{x})p(\mathbf{y})p(\mathbf{z}) dx dy dz = \\ &= \int p(\mathbf{x}) dx \int p(\mathbf{y}) dy \int p(\mathbf{z}) dz \end{aligned}$$

\Rightarrow integrals decompose into easier ones



Smoothness + decomposability = tractable MAR

Forward pass evaluation for MAR

\Rightarrow linear in circuit size!

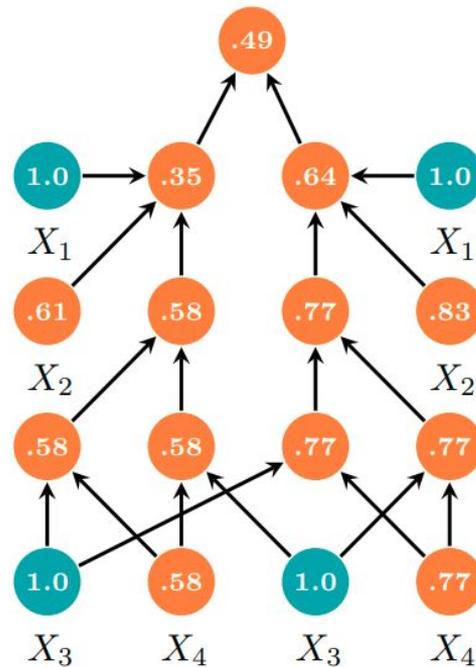
E.g. to compute $p(x_2, x_4)$:

■ leafs over X_1 and X_3 output $Z_i = \int p(x_i) dx_i$

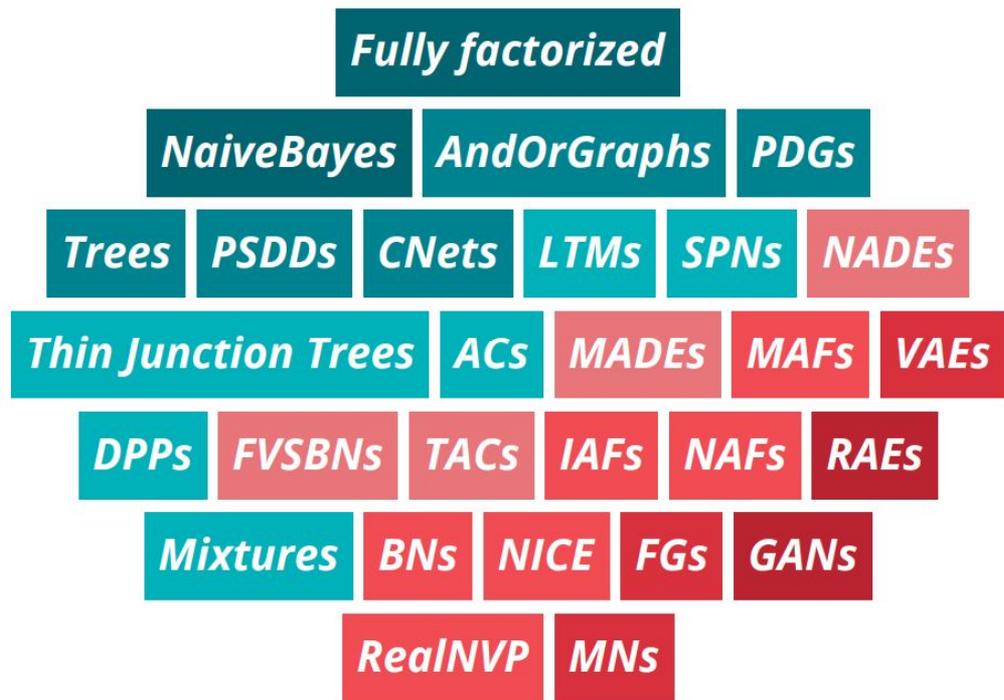
\Rightarrow for normalized leaf distributions: **1.0**

■ leafs over X_2 and X_4 output **EVI**

■ feedforward evaluation (bottom-up)

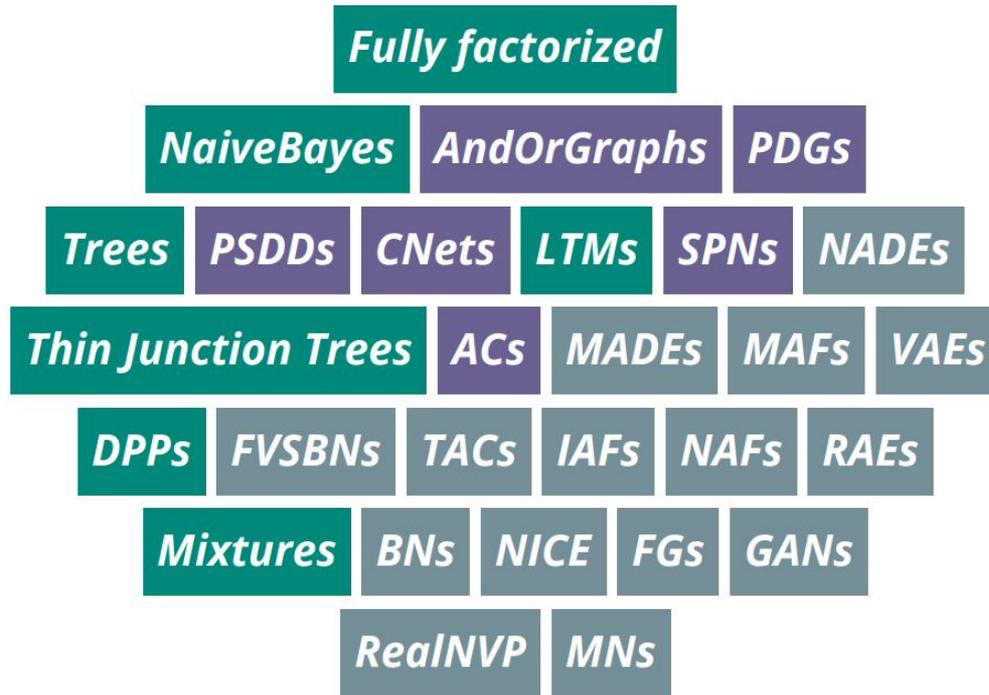


\mathcal{P}		\mathcal{Q} :								
		MAR	CON	MOM	MAP	MMAP	ENT	DIV	EXP	
		marginal queries	conditional queries	moments (mean,...)	maximum a posteriori	marginal MAP	entropy	divergences (KLD,...)	expected predictions	
smoothness	SMO	✓	✓	✓	✗	✗	✓	✓	✓	
decomposability	DEC	✓	✓	✓	✗	✗	✗	✗	✗	
consistency	CON	✗	✗	✗	✓	✓	✗	✗	✗	
determinism	DET	✗	✗	✗	✓	✗	✗	✗	✗	
marginal determinism	MAR-DET	✗	✗	✗	✗	✓	✓	✓	✗	
structured decomposability	STR-DEC	✗	✗	✗	✗	✗	✓	✗	✗	
paired str. decomposability	P-STR-DEC	✗	✗	✗	✗	✗	✗	✓	✓	



tractability is a spectrum

	<i>smooth</i>	<i>dec.</i>	<i>det.</i>	<i>str.dec.</i>
Arithmetic Circuits (ACs) [Darwiche 2003]	✓	✓	✓	✗
Sum-Product Networks (SPNs) [Poon et al. 2011]	✓	✓	✗	✗
Cutset Networks (CNets) [Rahman et al. 2014]	✓	✓	✓	✗
Probabilistic Decision Graphs [Jaeger 2004]	✓	✓	✓	✓
(Affine) ADDs [Hoey et al. 1999; Sanner et al. 2005]	✓	✓	✓	✓
AndOrGraphs [Dechter et al. 2007]	✓	✓	✓	✓
PSDDs [Kisa et al. 2014a]	✓	✓	✓	✓

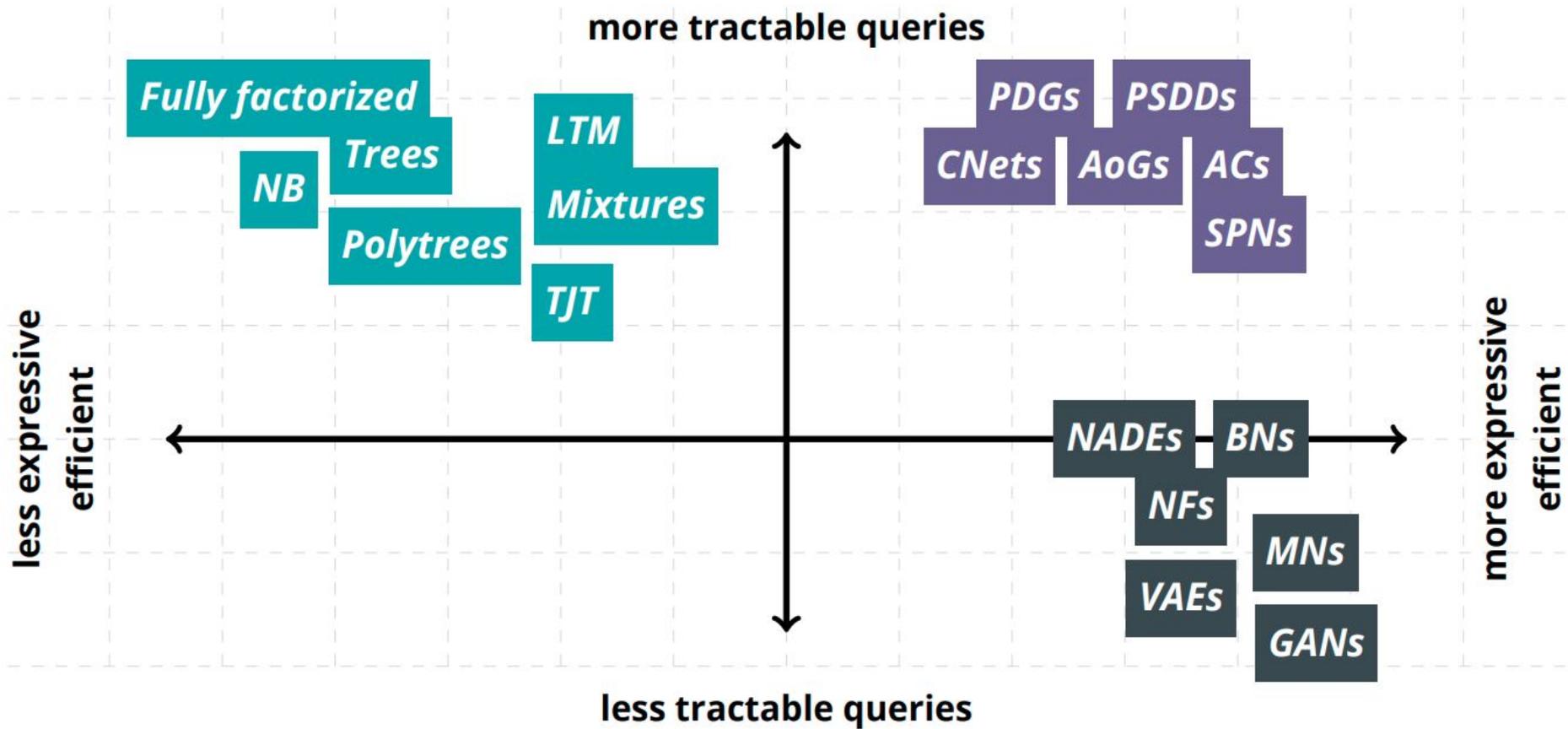


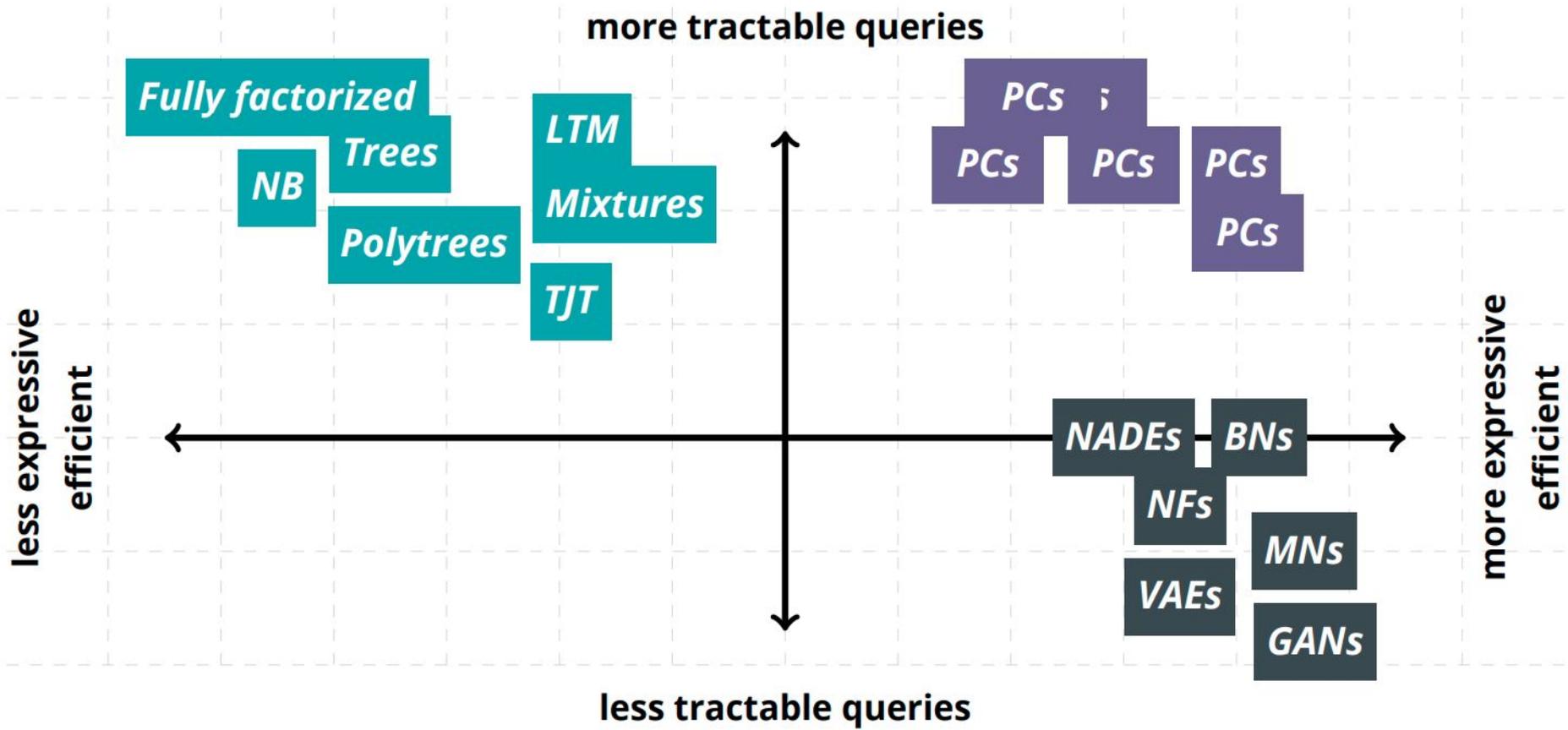
Expressive* models without *compromises

How expressive are probabilistic circuits?

density estimation benchmarks

dataset	best circuit	BN	MADE	VAE	dataset	best circuit	BN	MADE	VAE
<i>nlcs</i>	-5.99	-6.02	-6.04	-5.99	<i>dna</i>	-79.88	-80.65	-82.77	-94.56
<i>msnbc</i>	-6.04	-6.04	-6.06	-6.09	<i>kosarek</i>	-10.52	-10.83	-	-10.64
<i>kdd</i>	-2.12	-2.19	-2.07	-2.12	<i>msweb</i>	-9.62	-9.70	-9.59	-9.73
<i>plants</i>	-11.84	-12.65	-12.32	-12.34	<i>book</i>	-33.82	-36.41	-33.95	-33.19
<i>audio</i>	-39.39	-40.50	-38.95	-38.67	<i>movie</i>	-50.34	-54.37	-48.7	-47.43
<i>jester</i>	-51.29	-51.07	-52.23	-51.54	<i>webkb</i>	-149.20	-157.43	-149.59	-146.9
<i>netflix</i>	-55.71	-57.02	-55.16	-54.73	<i>cr52</i>	-81.87	-87.56	-82.80	-81.33
<i>accidents</i>	-26.89	-26.32	-26.42	-29.11	<i>c20ng</i>	-151.02	-158.95	-153.18	-146.9
<i>retail</i>	-10.72	-10.87	-10.81	-10.83	<i>bbc</i>	-229.21	-257.86	-242.40	-240.94
<i>pumbs*</i>	-22.15	-21.72	-22.3	-25.16	<i>ad</i>	-14.00	-18.35	-13.65	-18.81





Want to learn more?

Tutorial (3h)

Probabilistic Circuits

**Inference
Representations
Learning
Theory**

Antonio Vergari
University of California, Los Angeles

Robert Peharz
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YooJung Choi
University of California, Los Angeles

Guy Van den Broeck
University of California, Los Angeles

September 14th, 2020 - Ghent, Belgium - ECML-PKDD 2020

<https://youtu.be/2RAG5-L9R70>

Overview Paper (80p)

Probabilistic Circuits: A Unifying Framework for Tractable Probabilistic Models*

YooJung Choi
Antonio Vergari
Guy Van den Broeck
Computer Science Department
University of California
Los Angeles, CA, USA

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2.1	Probabilistic Models	5
2.2	Probabilistic Queries	6
2.3	Tractable Probabilistic Inference	8
2.4	Properties of Tractable Probabilistic Models	9

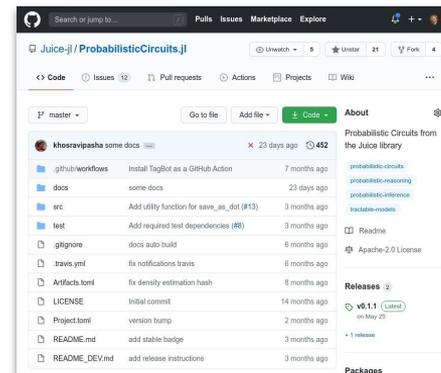
<http://starai.cs.ucla.edu/papers/ProbCirc20.pdf>

Training PCs in Julia with Juice.jl



Training maximum likelihood parameters of probabilistic circuits

```
julia> using ProbabilisticCircuits;
julia> data, structure = load(...);
julia> num_examples(data)
17,412
julia> num_edges(structure)
270,448
julia> @btime estimate_parameters(structure , data);
63 ms
```



Custom SIMD and CUDA kernels to parallelize over layers and training examples.

Probabilistic circuits seem awfully general.

*Are all tractable probabilistic models
probabilistic circuits?*



Determinantal Point Processes (DPPs)

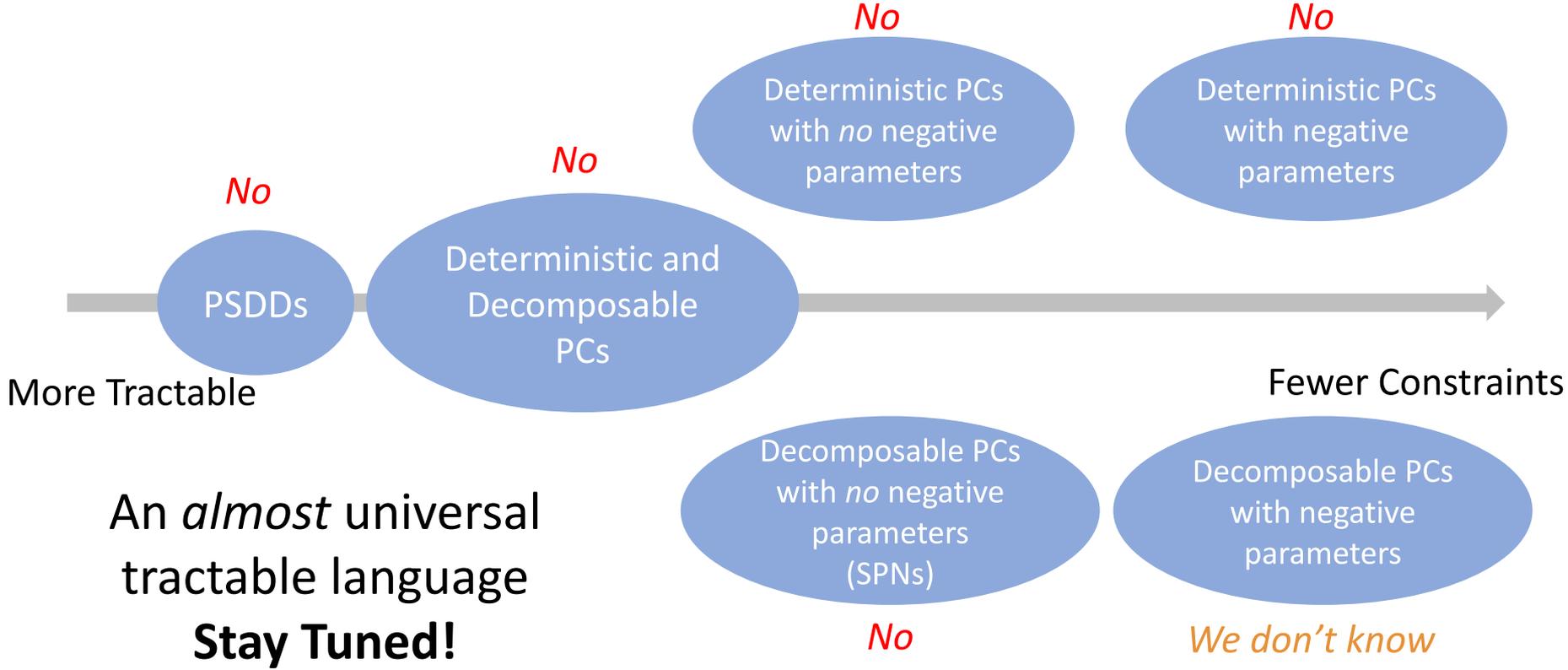
DPPs are models where probabilities are specified by (sub)determinants

$$L = \begin{bmatrix} \mathbf{1} & 0.9 & \mathbf{0.8} & 0 \\ 0.9 & 0.97 & 0.96 & 0 \\ \mathbf{0.8} & 0.96 & \mathbf{1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Pr_L(X_1 = \mathbf{1}, X_2 = 0, X_3 = \mathbf{1}, X_4 = 0) = \frac{1}{\det(L + I)} \det(L_{\{1,2\}})$$

Computing marginal probabilities is *tractable*.

We cannot tractably represent DPPs with classes of PCs ... yet



The AI Dilemma



Pure Logic

Pure Learning

The AI Dilemma



Pure Logic

Pure Learning

- Slow thinking: deliberative, cognitive, model-based, extrapolation
- Amazing achievements until this day
- “*Pure logic is brittle*”
noise, uncertainty, incomplete knowledge, ...



The AI Dilemma



Pure Logic

Pure Learning

- Fast thinking: instinctive, perceptive, model-free, interpolation
- Amazing achievements recently
- “*Pure learning is brittle*”

bias, algorithmic fairness, interpretability, explainability, adversarial attacks, unknown unknowns, calibration, verification, missing features, missing labels, data efficiency, shift in distribution, general robustness and safety fails to incorporate a sensible model of the world



Pure Logic Probabilistic World Models Pure Learning

A New Synthesis of
Learning and Reasoning

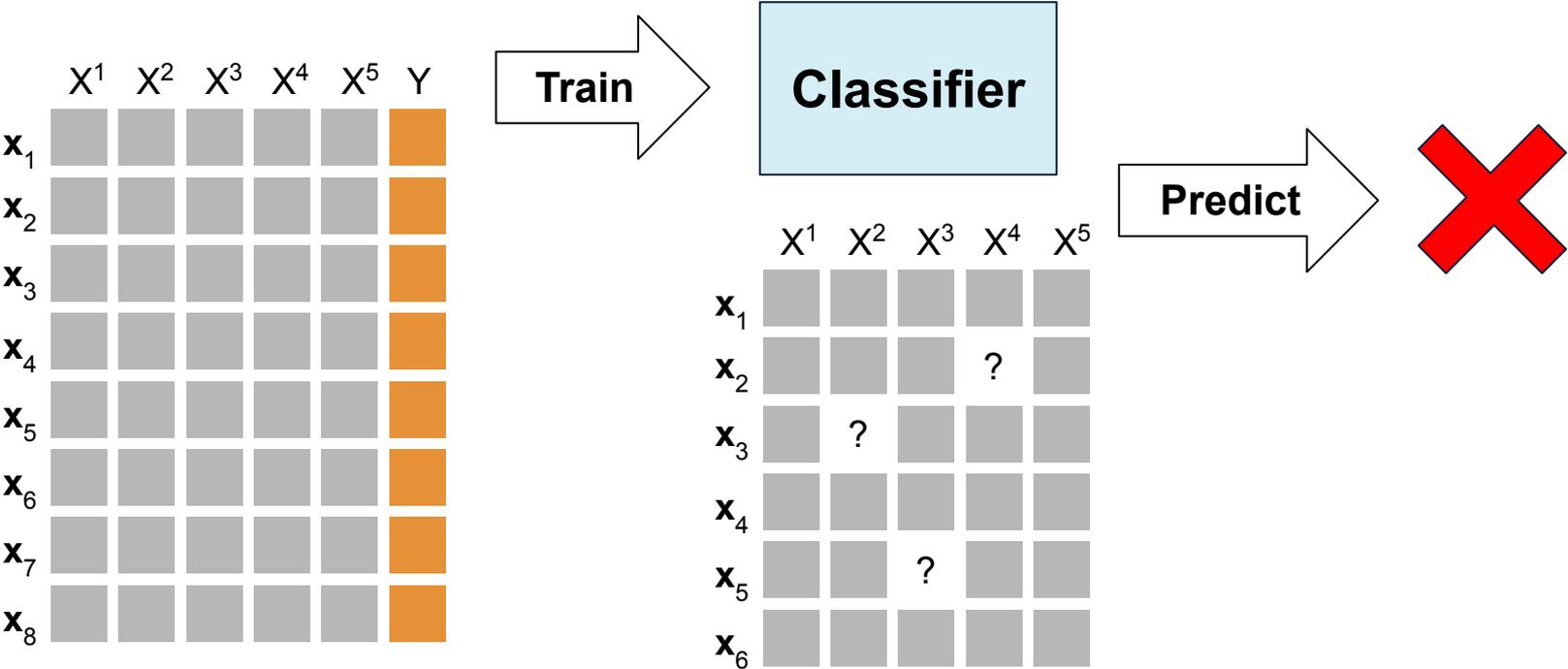
- “*Pure learning is brittle*”

bias, **algorithmic fairness**, interpretability, **explainability**, adversarial attacks, unknown unknowns, calibration, verification, **missing features**, missing labels, data efficiency, shift in distribution, general robustness and safety

fails to incorporate a sensible model of the world



Prediction with Missing Features



Test with missing features

Expected Predictions

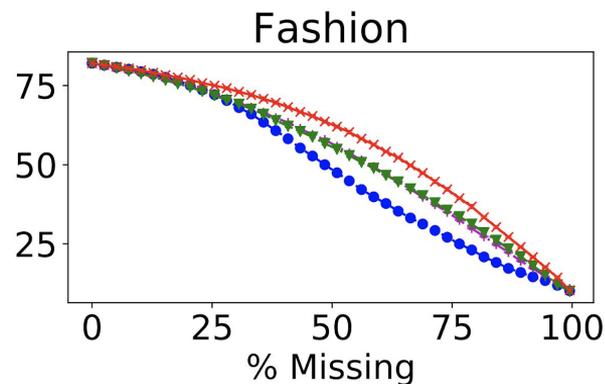
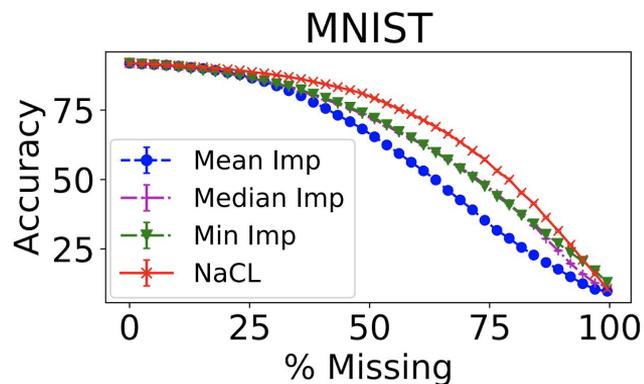
Consider **all possible complete inputs** and **reason** about the *expected* behavior of the classifier

$$\mathbb{E}_{\mathbf{x}^m \sim p(\mathbf{x}^m | \mathbf{x}^o)} \left[f(\mathbf{x}^m \mathbf{x}^o) \right]$$

\mathbf{x}^o = observed features
 \mathbf{x}^m = missing features

Experiment:

- $f(x)$ =
logistic regres.
- $p(x)$ =
naive Bayes



What about complex feature distributions?

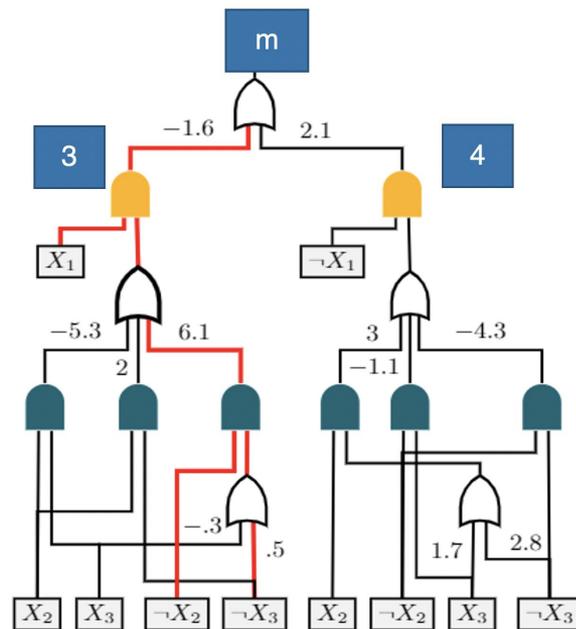
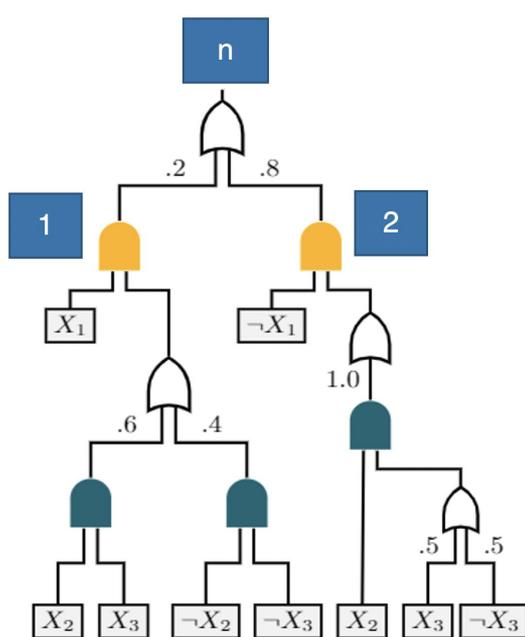
- feature distribution is a probabilistic circuits
- classifier is a compatible regression circuit



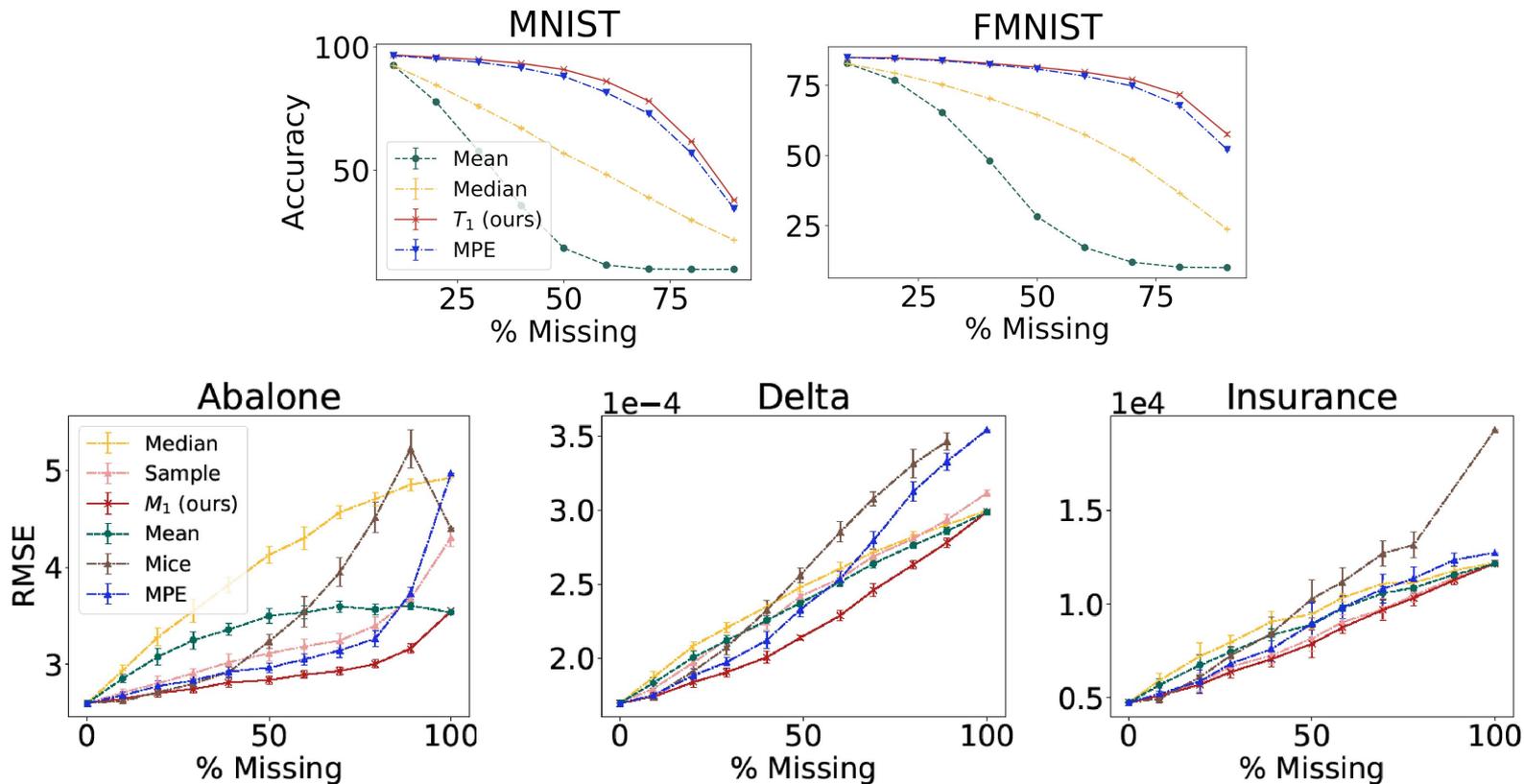
Recursion that
“breaks down”
the computation.

Expectation of
function m w.r.t. dist. n ?

Solve subproblems:
 $(1,3)$, $(1,4)$, $(2,3)$, $(2,4)$



Probabilistic Circuits for Missing Data



ADV inference in Julia with Juice.jl



```
using ProbabilisticCircuits
pc = load_prob_circuit(zoo_psdd_file("insurance.psdd"));
rc = load_logistic_circuit(zoo_lc_file("insurance.circuit"), 1);
```

Is the predictive model biased by gender?

```
groups = make_observations([[ "male" ], [ "female" ]])
exps, _ = Expectation(pc, rc, groups);
println("Female   : \$ $(exps[2])");
println("Male     : \$ $(exps[1])");
println("Diff      : \$ $(exps[2] - exps[1])");
Female   : $ 14170.125469335406
Male     : $ 13196.548926381849
Diff     : $ 973.5765429535568
```

Model-Based Algorithmic Fairness: FairPC

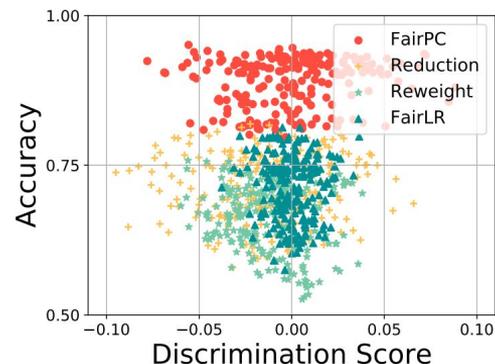
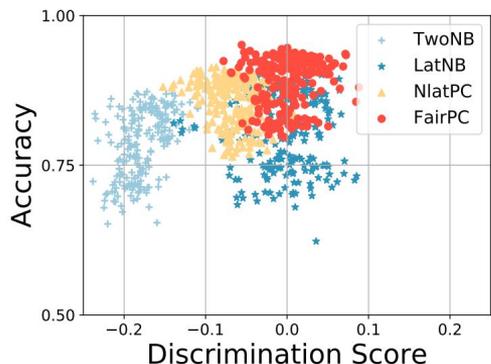
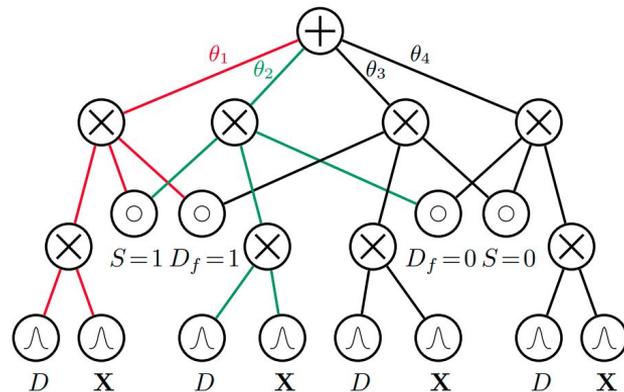
Learn classifier given

- features S and X
- training labels/decisions D

Group fairness by demographic parity:

Fair decision D_f should be independent of the sensitive attribute S

Discover the latent fair decision D_f by learning a PC.

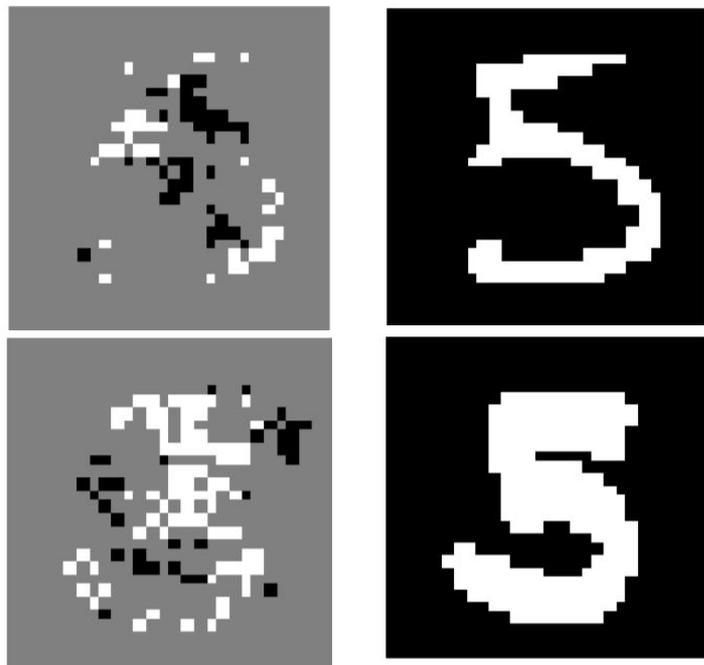


Probabilistic Sufficient Explanations

Goal: explain an instance of classification (a specific prediction)

Explanation is a subset of features, s.t.

1. The explanation is “probabilistically sufficient”
Under the feature distribution, given the explanation, the classifier is likely to make the observed prediction.
2. It is minimal and “simple”





Pure Logic Probabilistic World Models Pure Learning

A New Synthesis of Learning and Reasoning

“Pure learning is brittle”

bias, **algorithmic fairness**, interpretability, **explainability**, adversarial attacks, unknown unknowns, calibration, verification, **missing features**, missing labels, data efficiency, shift in distribution, general robustness and safety

We need to incorporate a sensible probabilistic model of the world

Probabilistic Programs



What are probabilistic programs?

```
let x = flip 0.5 in
let y = flip 0.7 in
let z = x || y in
let w = if z then
  my_func(x,y)
else
  ...
in
observe(z);
```

means “flip a coin, and
output true with probability $\frac{1}{2}$ ”

Standard (functional) programming
constructs: let, if, ...

means
“reject this execution if z is not true”

Why Probabilistic Programming?

PPLs are proliferating



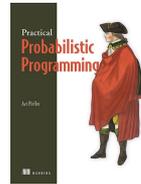
Edward



HackPPL



Stan



Figaro

Venture, Church, IBAL, WebPPL, Infer.NET, Tensorflow Probability, ProbLog, PRISM, LPADs, CLogic, CLP(BN), ICL, PHA, Primula, Storm, Gen, PRISM, PSI, Bean Machine, etc. ... *and many many more*

Programming languages are humanity's biggest knowledge representation achievement!

Programs should be AI models

Dice probabilistic programming language

<http://dicelang.cs.ucla.edu/>



Dice

The dice probabilistic programming language

About GitHub

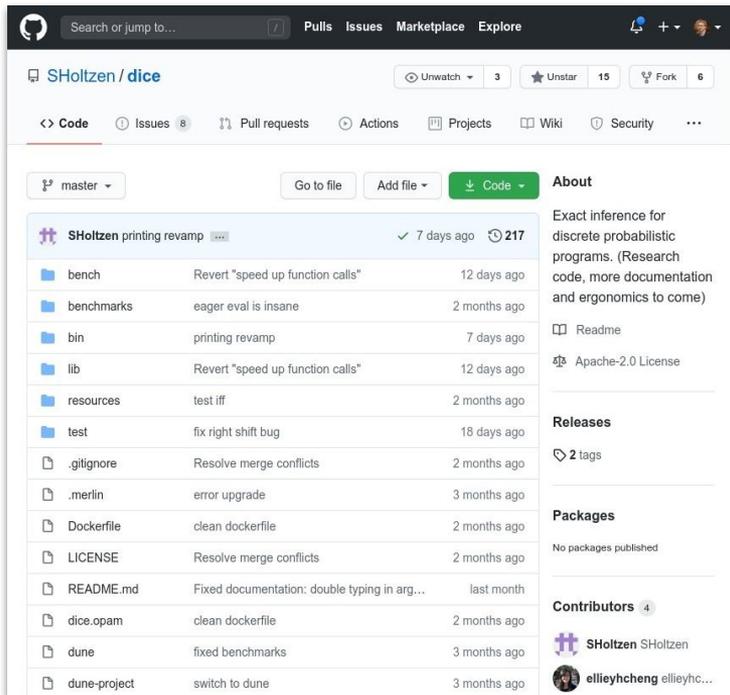
`dice` is a probabilistic programming language focused on fast exact inference for discrete probabilistic programs. For more information on `dice`, see the [about page](#).

Below is an online `dice` code demo. To run the example code, press the "Run" button.

```
1 fun sendChar(key: int(2), observation: int(2)) {
2   let gen = discrete(0.5, 0.25, 0.125, 0.125) in // sample a FooLang character
3   let enc = key + gen in // encrypt the character
4   observe observation == enc
5 }
6
7 // sample a uniform random key: A=0, B=1, C=2, D=3
8
9 let key = discrete(0.25, 0.25, 0.25, 0.25) in
10
11 // observe the ciphertext CCCC
12 let tmp = sendChar(key, int(2, 2)) in
13 let tmp = sendChar(key, int(2, 2)) in
14 let tmp = sendChar(key, int(2, 2)) in
15 let tmp = sendChar(key, int(2, 2)) in
16
17 key
```

Run

<https://github.com/SHoltzen/dice>



SHoltzen / dice

Code Issues 8 Pull requests Actions Projects Wiki Security

master

Go to file Add file Code

SHoltzen printing revamp 7 days ago 217

bench	Revert "speed up function calls"	12 days ago
benchmarks	eager eval is insane	2 months ago
bin	printing revamp	7 days ago
lib	Revert "speed up function calls"	12 days ago
resources	test iff	2 months ago
test	fix right shift bug	18 days ago
.gitignore	Resolve merge conflicts	2 months ago
.merlin	error upgrade	3 months ago
Dockerfile	clean dockerfile	2 months ago
LICENSE	Resolve merge conflicts	2 months ago
README.md	Fixed documentation: double typing in arg...	last month
dice.opam	clean dockerfile	2 months ago
dune	fixed benchmarks	3 months ago
dune-project	switch to dune	3 months ago

About

Exact inference for discrete probabilistic programs. (Research code, more documentation and ergonomics to come)

Readme Apache-2.0 License

Releases 2 tags

Packages No packages published

Contributors 4

SHoltzen SHoltzen ellieyhcheng ellieyh...

Why should I care?

Better abstraction than probabilistic graphical models:

- Beyond variable-level dependencies (contextual)
- modularity through functions
reuse (cf. relational graphical models)
- intuitive language for local structure; arithmetic
- data structures
- first-class observations

First-Class Observations

```
1 fun EncryptChar(key:int, obs:char):Bool {
2   let randomChar = ChooseChar() in
3   let ciphertext = (randomChar + key) % 26 in
4   let _ = observe ciphertext = obs in
5   true}
6 let k = UniformInt(0, 25) in
7 let _ = EncryptChar(k, 'H') in ...
8 let _ = EncryptChar(k, 'D') in k
```

Frequency Analyzer for a Caesar cipher in Dice

Probabilistic Program Inference

Key ingredient: **factorization**
.... aka the product nodes

```
1 let x = flip1 0.1 in
2 let y = if x then flip2 0.2 else
3   flip3 0.3 in
4 let z = if y then flip4 0.4 else
5   flip5 0.5 in z
```

$$\underbrace{0.1}_{x=T} \cdot \underbrace{0.2}_{y=T} \cdot \underbrace{0.4}_{z=T} + \underbrace{0.1}_{x=T} \cdot \underbrace{0.8}_{y=F} \cdot \underbrace{0.5}_{z=T} + \underbrace{0.9}_{x=F} \cdot \underbrace{0.3}_{y=T} \cdot \underbrace{0.4}_{z=T} + \underbrace{0.9}_{x=F} \cdot \underbrace{0.7}_{y=F} \cdot \underbrace{0.5}_{z=T}$$

$$\rightarrow \underbrace{0.1}_{x=T} \cdot \left(\underbrace{0.2}_{y=T} \cdot \underbrace{0.4}_{z=T} + \underbrace{0.8}_{y=F} \cdot \underbrace{0.5}_{z=T} \right) + \underbrace{0.9}_{x=F} \cdot \left(\underbrace{0.3}_{y=T} \cdot \underbrace{0.4}_{z=T} + \underbrace{0.7}_{y=F} \cdot \underbrace{0.5}_{z=T} \right)$$

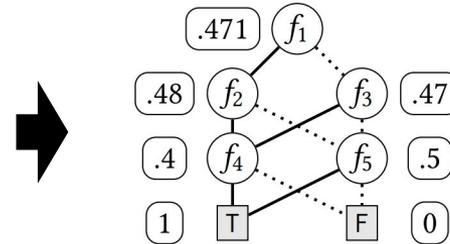
Symbolic Compilation in Dice

- Construct Boolean formula
- Satisfying assignments \approx paths
- Variables are flips
- Associate weights with flips
- Compile factorized circuit

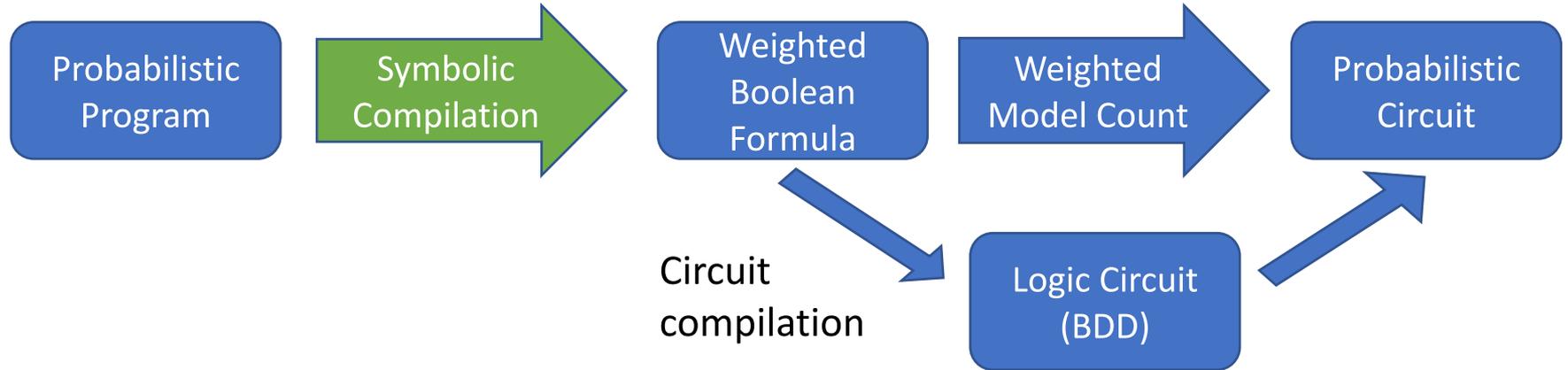
```
1 let x = flip1 0.1 in
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5   flip5 0.5 in z
```

$$\underbrace{0.1}_{x=T} \cdot \underbrace{0.2}_{y=T} \cdot \underbrace{0.4}_{z=T} + \underbrace{0.1}_{x=T} \cdot \underbrace{0.8}_{y=F} \cdot \underbrace{0.5}_{z=T} + \underbrace{0.9}_{x=F} \cdot \underbrace{0.3}_{y=T} \cdot \underbrace{0.4}_{z=T} + \underbrace{0.9}_{x=F} \cdot \underbrace{0.7}_{y=F} \cdot \underbrace{0.5}_{z=T}$$

$$\Rightarrow f_1 f_2 f_4 \vee f_1 \bar{f}_2 f_5 \vee \bar{f}_1 f_3 f_4 \vee \bar{f}_1 \bar{f}_3 f_5$$

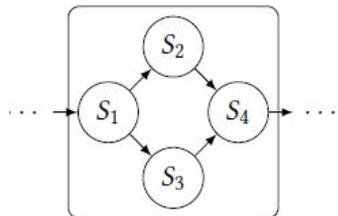


Symbolic Compilation to Probabilistic Circuits



State of the art for discrete probabilistic program inference!

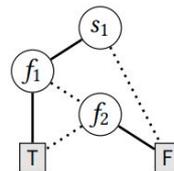
Factorized Inference in Dice



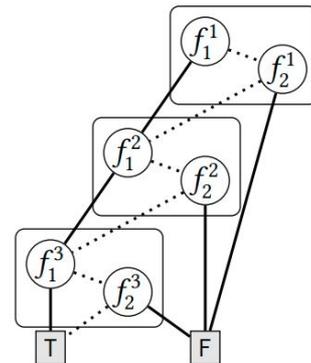
(a) Network diagram.

```
1 fun diamond( $s_1$ :Bool):Bool {  
2   let route = flip1 0.5 in  
3   let  $s_2$  = if route then  $s_1$  else F in  
4   let  $s_3$  = if route then F else  $s_1$  in  
5   let drop = flip2 0.0001 in  
6    $s_2 \vee (s_3 \wedge \neg \text{drop})$ }  
7 diamond(diamond(diamond(T)))
```

(b) Probabilistic program defining the network.



(c) diamond function.



(d) Final BDD.

Network Verification

PPL benchmarks from PL community

Benchmark	Psi (ms)	DP (ms)	Dice (ms)	# Paths	BDD Size
Grass	167	57	1.0	9.5×10^1	15
Burglar Alarm	98	10	1.1	2.5×10^2	11
Coin Bias	94	23	1.0	4	13
Noisy Or	81	152	1.0	1.6×10^4	35
Evidence1	48	32	1.0	9	5
Evidence2	59	28	1.0	9	6
Murder Mystery	193	75	1.0	1.6×10^1	6

Scalable Inference

Benchmark	Psi (ms)	DP (ms)	Dice (ms)	# Parameters	# Paths	BDD Size
Cancer [48]	772	46	1.0	10	1.1×10^3	28
Survey [73]	2477	152	2.0	21	1.3×10^4	73
Alarm [5]	X	X	9.0	509	1.0×10^{36}	1.3×10^3
Insurance [7]	X	X	75.0	984	1.2×10^{40}	1.0×10^5
Hepar2 [63]	X	X	54.0	1453	2.9×10^{69}	1.3×10^3
Hailfinder [1]	X	X	526.0	2656	2.0×10^{76}	6.5×10^4
Pigs	X	X	32.0	5618	7.3×10^{492}	35
Water [43]	X	X	2926.0	1.0×10^4	3.2×10^{54}	5.1×10^4
Munin [3]	X	X	1945.0	8.1×10^5	2.1×10^{1622}	1.1×10^4

Conclusions

- Are we already in the age of computational abstractions?
- **Probabilistic circuits** for learning deep tractable probabilistic models
- **Probabilistic programs** as the new probabilistic knowledge representation language
- Two computational abstractions go hand in hand



Thanks

*My students/postdoc who did
the real work are graduating.*

*There are some awesome people on the
academic job market!*