Higher-Order Situation Theory in Artificial Intelligence

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8th International Conference on Agents and Artificial Intelligence
ICAART 2016

Rome, Italy, 24–26 February, 2016

Origins and Present of Situation Theory (SitT)

- Barwise [1] (1981) is the most influential and debated, early work on SitT
- Barwise and Perry [2] (1983): a strategy of developing SitT
- Devlin [5, 6] (2008) an intuitive introduction to informal SitT
- Seligman and Moss [14] (2011): a model theory of SitT
- Loukanova [11] (2014) is an intro to the mathematics of set-theoretical (non-well founded) foundations of SitT
  - information in context, w.r.t. agents
- This work: new type-theoretic approach to formal syntax and semantics of typed information
  - pure variables (for abstractions) and recursion variables (for memory)
  - theory of recursion on relations and functions
  - recursion terms
  - generalized variables with constraints

Sets of basic situation theoretical objects

- Primitive individuals: $\mathcal{A}_{\mathrm{IND}} = \{a, b, c, \ldots\}$
- Space-time locations: $\mathcal{A}_{\mathrm{LOC}} = \{l, l_0, l_1, \ldots\}$
  associated with some space and time relations, e.g.:

$$l_i \prec l_j \qquad \text{(time precedence)}$$
$$l_i \circ l_j \qquad \text{(time overlapping)}$$
$$l_i \diamond l_j \qquad \text{(space overlapping)}$$
$$l_i \subseteq_t l_j \qquad \text{(time inclusion)}$$
$$l_i \subseteq_s l_j \qquad \text{(space inclusion)}$$
$$l_i \subseteq l_j \qquad \text{(space-time inclusion)}$$

- Primitive relations: $\mathcal{A}_{\mathrm{REL}} = \{r_0, r_1, \ldots\}$

Primitive (basic) types

$$B_{\text{TYPE}} = \{ \text{IND}, \text{REL}, \text{ArgR}, \text{LOC}, \text{POL}, \tag{2a}$$

$$\text{INFON}, \text{SIT}, \text{PROP}, \text{PAR}, \text{TYPE}, \models \} \tag{2b}$$

- IND: primitive and complex individuals;
- REL: primitive and complex relations;
- ArgR: primitive and complex argument roles;
- LOC: space-time locations;
- POL: polarities 0 and 1;
- INFON: basic or complex information units;
- SIT: situations;
- PROP: basic or complex propositions;
- PAR: primitive and complex parameters;
- TYPE: basic and complex types;

- $\models$ is a special type called "supports" ("holds") used in propositions that a situation s and an infon σ are of the type "supports", i.e., "s supports σ":

$$(s \models \sigma) \qquad \text{(a proposition)}$$
$$s \models \sigma \qquad \text{(a verified proposition)}$$

- Primitive and complex types $\mathcal{T}_{\mathrm{TYPE}}$

$$B_{\mathrm{TYPE}} \subseteq \mathcal{T}_{\mathrm{TYPE}} \tag{4}$$

- basic argument roles: $\mathcal{BA}_{\mathrm{ArgR}}$, e.g., $\mathcal{BA}_{\mathrm{ArgR}} = \{\, \rho_1, \ldots, \rho_m \,\}$;
  basic and complex argument roles: $\mathcal{BA}_{\mathrm{ArgR}} \subseteq \mathcal{A}_{\mathrm{ArgR}}$
- A set of argument roles is assigned to the primitive relations and types by a function *ArgR*. I.e.:
- for every $\gamma \in \mathcal{A}_{\mathrm{REL}} \cup B_{\mathrm{TYPE}}$

$$ArgR(\gamma) = \{\langle arg_1, T_1\rangle, \ldots, \langle arg_n, T_n\rangle\} \quad (5)$$

$$\equiv \{\, T_1 : arg_1, \ldots, T_n : arg_n\} \quad (n \geq 0) \quad (6)$$

  where $arg_1, \ldots, arg_n \in \mathcal{A}_{\mathrm{ArgR}}$,
  $T_1, \ldots, T_n \in \mathcal{P}(\mathcal{T}_{\mathrm{TYPE}})$ are sets of types (basic or complex).
- The objects $arg_1, \ldots, arg_n$ are called the argument roles or argument slots of γ.
- $T_1, \ldots, T_n$ are specific for γ and are called the appropriateness constraints of the argument roles of γ.

Relations and Types with Argument Roles

- Each relation is associated with a set *ArgR* of argument roles

$$ArgR(smile) = \{T_a : smiler\} \tag{7a}$$

$$ArgR(read) = \{T_{a_1} : reader,\ T_m : read\text{-}ed,\ T_{a_2} : readee\} \tag{7b}$$

$$ArgR(read_1) = \{T_a : reader, T_o : read\text{-}ed\}\} \tag{7c}$$

$$ArgR(give) = \{T_a : giver,\ T_r : receiver,\ T_g : given\} \tag{7d}$$

- Each type is associated with a set *ArgR* of argument roles, e.g., for the "supports" type $\models$ of situations and infons:

$$ArgR(\models) = \{\mathrm{SIT} : arg_{\mathrm{SIT}},\ \mathrm{INFON} : arg_{\mathrm{INFON}}\}. \tag{8}$$

Primitive parameters

- Typed primitive parameters (sometimes called indeterminates):

$$\mathcal{P}_{\mathrm{IND}} = \{\dot{a}, \dot{b}, \dot{c}, \ldots\}, \tag{9a}$$

$$\mathcal{P}_{\mathrm{LOC}} = \{\dot{l}_0, \dot{l}_1, \ldots\}, \tag{9b}$$

$$\mathcal{P}_{\mathrm{REL}} = \{\dot{r}_0, \dot{r}_1, \ldots\}, \tag{9c}$$

$$\mathcal{P}_{\mathrm{POL}} = \{\dot{i}_0, \dot{i}_1, \ldots\}, \tag{9d}$$

$$\mathcal{P}_{\mathrm{SIT}} = \{\dot{s}_0, \dot{s}_1, \ldots\}. \tag{9e}$$

We will define complex objects recursively

- Infons
- states
- events
- situations
- propositions
- situated propositions
- complex relations
- complex types
- restricted parameters

Definition (Basic Infons)

A basic infon is every tuple $\langle \gamma, \theta, \tau, i \rangle$, where

- $\gamma \in \mathcal{R}_{\mathrm{REL}}$ is a relation (primitive or complex)

$$ArgR(\gamma) = \{\langle arg_1, T_1 \rangle, \ldots, \langle arg_n, T_n \rangle\} \quad (n \geq 0) \tag{10a}$$

$$T_1, \ldots, T_n \in \mathcal{P}(\mathcal{T}_{\mathrm{TYPE}}) \tag{10b}$$

- θ is an argument filling for γ, i.e.:

$$\theta = \{\langle arg_1, \xi_1 \rangle, \ldots, \langle arg_n, \xi_n \rangle\}, \tag{11}$$

for $\xi_1, \ldots, \xi_n$ that satisfy the type constraints over γ:

$$T_1 : \xi_1, \ldots, T_n : \xi_n \tag{12}$$

- $\mathrm{LOC} : \tau$ (basic or complex), $\quad \mathrm{POL} : i, \; i \in \{0, 1\}$,

Definition (Infons)

The class $\mathcal{I}_{INF}$ of infons has basic and complex infons:

$$\mathcal{BI}_{INF} \subset \mathcal{I}_{INF}$$

- Complex infons (for representation of conjunctive, disjunctive, and negated information), e.g.:

  For any infons $\sigma_1, \sigma_2, \sigma \in \mathcal{I}_{INF}$,

$$\langle \wedge, \sigma_1, \sigma_2 \rangle \in \mathcal{I}_{INF} \tag{13a}$$

$$\langle \vee, \sigma_1, \sigma_2 \rangle \in \mathcal{I}_{INF} \tag{13b}$$

$$\langle \neg, \sigma \rangle \in \mathcal{I}_{INF} \tag{13c}$$

- basic infons in linear notations:

$$\begin{aligned}\ll \gamma,\ & T_1 : \mathrm{arg}_1 : \xi_1, \ldots, \\ & T_n : \mathrm{arg}_n : \xi_n, \\ & \textsc{loc} : Loc : \tau,\ \textsc{pol} : Pol : i \gg\end{aligned} \tag{14}$$

$$\ll \gamma, arg_1 : \xi_1, \ldots, arg_n : \xi_n, Loc : \tau; Pol : i \gg \tag{15}$$

$$\ll \gamma, \xi_1, \ldots, \xi_n, \tau; i \gg \tag{16}$$

Example (infons in linear notations)

An infon can be specific or parametric, e.g.

- *a* reads *b* to *c* at the space-time location *l* (specific objects)

$$\begin{aligned} \ll read,\ & T_{a_1} : reader : a, \\ & T_m : read\text{-}ed : b, \\ & T_{a_2} : readee : c, \\ & \mathrm{LOC} : Loc : l;\ \mathrm{POL} : Pol : 1 \gg \end{aligned} \tag{17}$$

- *a* reads *b* to the unknown $\dot{c}$ at the unknown location $\dot{l}$

$$\begin{aligned} \ll read,\ & T_{a_1} : reader : a, && \text{(specific)} \\ & T_m : read\text{-}ed : b, && \text{(specific)} \\ & T_{a_2} : readee : \dot{c},\ \dot{l};\ : 1 \gg && \text{(parametric)} \end{aligned}$$

Example (infons in linear notations)

Other parametric infons, e.g.

- *a* reads
  (the unknown $\dot{b}$ to the unknown $\dot{c}$ at the unknown location $\dot{l}$)

$$\begin{aligned} \ll read,\ & T_{a_1} : reader : a, && \text{(specific)} \\ & T_m : read\text{-}ed : \dot{b}, && \text{(parametric)} \\ & T_{a_2} : readee : \dot{c},\ \dot{l};\ 1 \gg && \text{(parametric)} \end{aligned}$$

- the info that *a* either reads or does not — unknown polarity $\dot{p}$

$$\begin{aligned} \ll read,\ & T_{a_1} : reader : a, && \text{(specific)} \\ & T_m : read\text{-}ed : \dot{b},\ T_{a_2} : readee : \dot{c},\ \dot{l}; && \text{(parametric)} \\ & \dot{p} \gg && \text{(parametric)} \end{aligned}$$

Definition (Propositions)

Proposition is any tuple $\langle \mathrm{PROP}, \mathbb{T}, \theta \rangle$, where

- $\mathbb{T} \in \mathcal{T}_{\mathrm{TYPE}}$ is a type with a set of argument roles

$$ArgR(\mathbb{T}) = \{\langle arg_1, T_1 \rangle, \ldots, \langle arg_n, T_n \rangle\}, \quad n \geq 0 \tag{21}$$

- θ is an argument filling for $\mathbb{T}$, i.e.:

$$\theta = \{\langle arg_1, \xi_1 \rangle, \ldots, \langle arg_n, \xi_n \rangle\}, \tag{22}$$

for some objects $\xi_1, \ldots, \xi_n$ that satisfy the appropriateness type constraints of the type $\mathbb{T}$, i.e.:

$$T_1 : \xi_1, \ldots, T_n : \xi_n \tag{23}$$

Notation

$$\langle \mathbb{T}, \theta \rangle \equiv (\mathbb{T} : \theta) \tag{24a}$$
$$\equiv (\theta : \mathbb{T}) \tag{24b}$$
$$\equiv \langle \text{PROP}, \mathbb{T}, \theta \rangle \tag{24c}$$

- The variant notations (24a) and (24b) are used depending on context.
- The notation (24a) resemble the application operation.

Definition (Situated propositions)

- The type $\models$ ("supports"):

$$ArgR(\models) = \{\mathrm{SIT} : arg_{\mathrm{SIT}},\ \mathrm{INFON} : arg_{\mathrm{INFON}}\} \tag{25}$$

- *Situated proposition*:

$$\langle \mathrm{PROP}, \models, s, \sigma \rangle, \text{ where } s \in \mathcal{P}_{\mathrm{SIT}} \text{ and } \sigma \in \mathcal{I}_{\mathrm{INFON}} \tag{26}$$

Notation

$$\langle \models, s, \sigma \rangle \equiv (s \models \sigma) \tag{27a}$$
$$\equiv \langle \mathrm{PROP}, \models, s, \sigma \rangle \tag{27b}$$

Example (The proposition that s supports a positive infon)

$$(s \models \ll book, \textsc{ind} : arg : b, \tag{28a}$$

$$\textsc{loc} : Loc : l; \textsc{pol} : Pol : 1 \gg) \tag{28b}$$

Example (The proposition that s supports a negative infon)

$$(s \models \ll book, \textsc{ind} : arg : b, \tag{29a}$$

$$\textsc{loc} : Loc : l; \textsc{pol} : Pol : 0 \gg) \tag{29b}$$

Example (The situation s does not support a positive infon)

$$(s \not\models \ll book, \textsc{ind} : arg : b, \tag{30a}$$

$$\textsc{loc} : Loc : l; \textsc{pol} : Pol : 1 \gg) \tag{30b}$$

Example (The situation s does not support a negative infon)

$$(s \not\models \ll book, \textsc{ind} : arg : b, \tag{31a}$$

$$\textsc{loc} : Loc : l; \textsc{pol} : Pol : 0 \gg) \tag{31b}$$

Example (actual vs. fallible situations)

$$(s_1 \models \ll book, b, l; 1 \gg) \tag{32a}$$
$$(s_2 \models \ll book, b, l; 0 \gg) \tag{32b}$$

- In case that both propositions (32a), (32b) are true, (without being part of perspective environments) at least one of the situations s_1, s_2 is not actual, because of the shared location l.
- It may be that
  - s_1 is actual situation, corresponding to a part of the reality
  - s_2 is erroneous, i.e., "carries" wrong information
    E.g., s_2 can be a state of an informational entity.

Example (actual vs. fallible situations)

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- It may be that
  - s_1 is actual situation, corresponding to a part of the reality
  - s_2 is erroneous, i.e., "carries" wrong information
    E.g., s_2 can be a state of an informational entity.

Example (A situation s can "carry" partial information)

$$(s \not\models \ll book,\ b,\ l;\ 1 \gg) \tag{33a}$$

$$(s \not\models \ll book,\ b,\ l;\ 0 \gg) \tag{33b}$$

Both propositions (33a) and (33b) can be true.

Example (conjunctive information)

- a conjunctive infon in a proposition

$$(s \models \quad \ll \textit{smiles}, \ \text{IND} : \textit{arg} : a, \ \text{LOC} : \textit{Loc} : l; 1 \gg \quad (34a)$$
$$\wedge \ll \textit{animate}, \ \text{IND} : \textit{arg} : a, \ l_1; 1 \gg \quad (34b)$$
$$\wedge \ l \circ l_1) \quad (34c)$$

- a conjunctive proposition

$$(s \models \ \ll \textit{smiles}, \ \text{IND} : \textit{arg} : a, \ l; 1 \gg) \quad (35a)$$
$$\wedge (s \models \ \ll \textit{animate}, \ \text{IND} : \textit{arg} : a, \ l_1; 1 \gg) \quad (35b)$$
$$\wedge (l \circ l_1) \quad (35c)$$

- There is another way to present the information (34b) and (35b). More on this later.

Example (conjunctive information)

- a conjunctive infon in a proposition

$$(s \models \ll smiles,\ \textsc{ind} : arg : a,\ \textsc{loc} : Loc : l; 1 \gg \quad (34a)$$
$$\wedge \ll animate,\ \textsc{ind} : arg : a,\ l_1; 1 \gg \quad (34b)$$
$$\wedge\ l \circ l_1) \quad (34c)$$

- a conjunctive proposition

$$(s \models \ll smiles,\ \textsc{ind} : arg : a,\ l; 1 \gg) \quad (35a)$$
$$\wedge (s \models \ll animate,\ \textsc{ind} : arg : a,\ l_1; 1 \gg) \quad (35b)$$
$$\wedge (l \circ l_1) \quad (35c)$$

- There is another way to present the information (34b) and (35b). More on this later.

Example (conjunctive information)

- a conjunctive infon in a proposition

$$(s \models \ll \textit{smiles},\ \textsc{ind} : \textit{arg} : a,\ \textsc{loc} : \textit{Loc} : l; 1 \gg \quad (34a)$$
$$\wedge \ll \textit{animate},\ \textsc{ind} : \textit{arg} : a,\ l_1; 1 \gg \quad (34b)$$
$$\wedge\ l \circ l_1) \quad (34c)$$

- a conjunctive proposition

$$(s \models \ll \textit{smiles},\ \textsc{ind} : \textit{arg} : a,\ l; 1 \gg) \quad (35a)$$
$$\wedge (s \models \ll \textit{animate},\ \textsc{ind} : \textit{arg} : a,\ l_1; 1 \gg) \quad (35b)$$
$$\wedge (l \circ l_1) \quad (35c)$$

- There is another way to present the information (34b) and (35b). More on this later.

Example

- The propositional content of the sentence (36) might be expressed by the proposition (37a)–(37c), with some (great) approximation.

$$\text{The book } b \text{ is read} \tag{36}$$

$$(s \models \ \ll \textit{read},\ \textit{reader} : \dot{x},\ \textit{readed} : b,\ \textit{readee} : \dot{y},\ \textit{Loc} : l; 1 \gg \tag{37a}$$

$$\wedge \ll \textit{book},\ \textit{arg} : b,\ \textit{Loc} : l_1; 1 \gg) \tag{37b}$$

$$\wedge\ (l \subset l_1) \tag{37c}$$

(37b) and (37c) are presented as parts of the propositional content of (36). There are other ways to include this information (later).

Definition (Complex relations and appropriateness constraints)

- Let σ be a given infon, and $\{\xi_1, \ldots, \xi_n\}$ a set of parameters that occur in σ.
- Let, for each $i \in \{1, \ldots, n\}$, T_i be the union of the constraints over the argument roles filled up by ξ_i.
- Then $\lambda\{\xi_1, \ldots, \xi_n\}\sigma$ is a complex relation, with abstract argument roles denoted by $[\xi_1], \ldots, [\xi_n]$ and having $T_1, \ldots, T_n$ as appropriateness type constraints, respectively, i.e.:

$$\begin{aligned} &ArgR(\lambda\{\xi_1, \ldots, \xi_n\}\sigma) \\ &= \{\langle[\xi_1], T_1\rangle, \ldots, \langle[\xi_n], T_n\rangle\} \end{aligned} \quad (38)$$

Example (A complex infon)

$$\ll book,\ b,\ l_1;\ 0 \gg \quad (39a)$$
$$\land \ll writes,\ a,\ b,\ l_2;\ 1 \gg \quad (39b)$$
$$\land \ll book,\ b,\ l_3;\ 1 \gg \quad (39c)$$
$$\land\ l_1 \prec l_2\ \land\ l_2 \prec l_3 \quad (39d)$$

Example (A complex relation between $\dot{x}$, $\dot{y}$, and locations $\dot{l}_1$, $\dot{l}_2$, $\dot{l}_3$)

$$\lambda\{\dot{x},\dot{y},\dot{l}_1,\dot{l}_2,\dot{l}_3\}\big[\ll book,\ \dot{y},\ \dot{l}_1;\ 0 \gg \quad (40a)$$
$$\land \ll writes,\ \dot{x},\ \dot{y},\ \dot{l}_2;\ 1 \gg \quad (40b)$$
$$\land \ll book,\ \dot{y},\ \dot{l}_3;\ 1 \gg \quad (40c)$$
$$\land\ \dot{l}_1 \prec \dot{l}_2\ \land\ \dot{l}_2 \prec \dot{l}_3\big] \quad (40d)$$

Example (A complex infon)

$$\ll book,\ b,\ l_1;\ 0 \gg \tag{39a}$$
$$\wedge \ll writes,\ a,\ b,\ l_2;\ 1 \gg \tag{39b}$$
$$\wedge \ll book,\ b,\ l_3;\ 1 \gg \tag{39c}$$
$$\wedge\ l_1 \prec l_2 \ \wedge\ l_2 \prec l_3 \tag{39d}$$

Example (A complex relation between $\dot{x}$, $\dot{y}$, and locations $\dot{l}_1$, $\dot{l}_2$, $\dot{l}_3$)

$$\lambda\{\dot{x}, \dot{y}, \dot{l}_1, \dot{l}_2, \dot{l}_3\}\big[\ll book,\ \dot{y},\ \dot{l}_1;\ 0 \gg \tag{40a}$$
$$\wedge \ll writes,\ \dot{x},\ \dot{y},\ \dot{l}_2;\ 1 \gg \tag{40b}$$
$$\wedge \ll book,\ \dot{y},\ \dot{l}_3;\ 1 \gg \tag{40c}$$
$$\wedge\ \dot{l}_1 \prec \dot{l}_2 \ \wedge\ \dot{l}_2 \prec \dot{l}_3\big] \tag{40d}$$

Definition (Complex types and appropriateness constraints)

- Let Θ be a given proposition, and $\{\xi_1, \dots, \xi_n\}$ a set of parameters that occur in Θ.
- Let, for each $i \in \{1, \dots, n\}$, T_i be the union of the constraints over the argument roles filled up by ξ_i.
- Then $\lambda\{\xi_1, \dots, \xi_n\}\Theta$ is a complex type, with abstract argument roles denoted by $[\xi_1], \dots, [\xi_n]$ and having $T_1, \dots, T_n$ as appropriateness type constraints, respectively, i.e.:

$$
\begin{aligned}
&ArgR(\lambda\{\xi_1, \dots, \xi_n\}\Theta) \\
&= \{\langle[\xi_1], T_1\rangle, \dots, \langle[\xi_n], T_n\rangle\}
\end{aligned}
\tag{41}
$$

Notation

Alternative classic notations for the complex types (corresponding to the set-theoretical comprehension):

$$\lambda\{\xi_1, \dots, \xi_n\}\Theta \equiv \left[T_1 : [\xi_1], \dots, T_n : [\xi_n] \mid \Theta \right] \tag{42a}$$

$$\lambda\{\xi_1, \dots, \xi_n\}\Theta \equiv \left[[\xi_1], \dots, [\xi_n] \mid \Theta \right] \tag{42b}$$

Notation

If Θ is a proposition, we sometimes use the following notation for types:

$$\{\, \xi_1, \dots, \xi_n \mid \Theta \,\} \equiv \text{TYPE} : \lambda\{\xi_1, \dots, \xi_n\}\Theta \tag{43}$$

Example (A proposition)

$$(s_1 \not\models \ll book,\ b,\ l_1;\ 0 \gg) \tag{44a}$$
$$\wedge\, (s_2 \models \ll writes,\ a,\ b,\ l_2;\ 1 \gg) \tag{44b}$$
$$\wedge\, (s_3 \models \ll book,\ b,\ l_3;\ 1 \gg) \tag{44c}$$
$$\wedge\, (l_1 \prec l_2 \prec l_3) \tag{44d}$$

Example (Complex type of objects $\dot{x}$, $\dot{y}$, and locations $\dot{l}_1$, $\dot{l}_2$, $\dot{l}_3$)

$$\lambda\{\dot{x}, \dot{y}, \dot{l}_1, \dot{l}_2, \dot{l}_3\}\big[(s_1 \not\models \ll book,\ \dot{y},\ \dot{l}_1;\ 0 \gg) \tag{45a}$$
$$\wedge\, (s_2 \models \ll writes,\ \dot{x},\ \dot{y},\ \dot{l}_2;\ 1 \gg) \tag{45b}$$
$$\wedge\, (s_3 \models \ll book,\ \dot{y},\ \dot{l}_3;\ 1 \gg) \tag{45c}$$
$$\wedge\, (\dot{l}_1 \prec \dot{l}_2 \prec \dot{l}_3)\big] \tag{45d}$$

Example (A proposition)

$$
\begin{aligned}
&(s_1 \not\models \ll book,\ b,\ l_1;\ 0 \gg) && \text{(44a)}\\
\land\ &(s_2 \models \ll writes,\ a,\ b,\ l_2;\ 1 \gg) && \text{(44b)}\\
\land\ &(s_3 \models \ll book,\ b,\ l_3;\ 1 \gg) && \text{(44c)}\\
\land\ &(l_1 \prec l_2 \prec l_3) && \text{(44d)}
\end{aligned}
$$

Example (Complex type of objects $\dot{x}$, $\dot{y}$, and locations $\dot{l}_1$, $\dot{l}_2$, $\dot{l}_3$)

$$
\begin{aligned}
\lambda\{\dot{x}, \dot{y}, \dot{l}_1, \dot{l}_2, \dot{l}_3\}\big[&(s_1 \not\models \ll book,\ \dot{y},\ \dot{l}_1;\ 0 \gg) && \text{(45a)}\\
\land\ &(s_2 \models \ll writes,\ \dot{x},\ \dot{y},\ \dot{l}_2;\ 1 \gg) && \text{(45b)}\\
\land\ &(s_3 \models \ll book,\ \dot{y},\ \dot{l}_3;\ 1 \gg) && \text{(45c)}\\
\land\ &(\dot{l}_1 \prec \dot{l}_2 \prec \dot{l}_3)\big] && \text{(45d)}
\end{aligned}
$$

Definition (Complex propositions)

- Let $\text{TYPE} : \lambda\{\xi_1, \dots, \xi_n\}\Theta$, and

$$ArgR(\lambda\{\xi_1, \dots, \xi_n\}\Theta) = \{\langle[\xi_1], T_1\rangle, \dots, \langle[\xi_n], T_n\rangle\} \quad (46)$$

- Let a_i be an object of all the types in T_i, i.e.,

$$T_i \equiv \bigcup\{ T_{i,1}, \dots T_{i,k_i} \} \quad (47a)$$

$$T_{i,1} : a_i, \dots, T_{i,k_i} : a_i, \quad \text{for } i = 1, \dots, n \quad (47b)$$

- Then we can form the proposition

$$\left(\lambda\{\xi_1, \dots, \xi_n\}\Theta, \theta\right) \quad (48)$$

where $\theta = \{\langle[\xi_1], a_1\rangle, \dots, \langle[\xi_n], a_n\rangle\}$.

Notation

$$(\lambda\{\xi_1, \ldots, \xi_n\}\Theta, \theta) \tag{49a}$$
$$\equiv (\lambda\{\xi_1, \ldots, \xi_n\}\Theta, \{T_1 : [\xi_1] : a_1, \ldots T_n : [\xi_n] : a_n\}) \tag{49b}$$
$$\equiv (\{T_1 : [\xi_1] : a_1, \ldots T_n : [\xi_n] : a_n\} : \lambda\{\xi_1, \ldots, \xi_n\}\Theta) \tag{49c}$$

Linear Notations

By assuming an order over the argument roles

$$(\lambda\{\xi_1, \ldots, \xi_n\}\Theta, \theta) \tag{50a}$$
$$\equiv (a_1, \ldots, a_n : \lambda\{\xi_1, \ldots, \xi_n\}\Theta) \tag{50b}$$
$$\equiv (\lambda\{\xi_1, \ldots, \xi_n\}\Theta \ \{a_1, \ldots, a_n\}) \quad \text{(reminds application)} \tag{50c}$$
$$\equiv (\lambda\{\xi_1, \ldots, \xi_n\}\Theta : a_1, \ldots, a_n) \quad \text{(reminds application)} \tag{50d}$$

Example (Complex proposition)

$$\Big(\lambda\{\dot{x}, \dot{y}, \dot{l}_1, \dot{l}_2, \dot{l}_3\} \Big[(s_1 \not\models \ll book,\ \dot{y},\ \dot{l}_1;\ 0 \gg) \tag{51a}$$

$$\wedge\, (s_2 \models \ll writes,\ \dot{x},\ \dot{y},\ \dot{l}_2;\ 1 \gg) \tag{51b}$$

$$\wedge\, (s_3 \models \ll book,\ \dot{y},\ \dot{l}_3;\ 1 \gg) \tag{51c}$$

$$\wedge\, (\dot{l}_1 \prec \dot{l}_2 \prec \dot{l}_3) \Big] \tag{51d}$$

$$: a, b, l_1, l_2, l_3 \Big) \tag{51e}$$

Definition (Complex restricted parameters)

Given that

- ξ is a parameter and $\Theta(\xi)$ is a proposition
- T is the set of the types that are constraints over the argument roles in $\Theta(\xi)$ that are filled up by ξ
- x is a parameter of type τ, i.e., $\tau : x$, and τ is compatible with the types (constraints) T,
- then $x^{\lambda\xi\,\Theta(\xi)}$ is a complex parameter of type τ, which is called a parameter restricted by the type $\lambda\xi\,\Theta(\xi)$.
- An object a can be anchored to the parameter $x^{\lambda\xi\,\Theta(\xi)}$
  $\iff$ a is of type τ, i.e., $\tau : a$,
  $T_i : a$, for each type $T_i \in T$,
  and $\lambda\xi\,\Theta(\xi) : a$, i.e., the proposition $\Theta(a)$ is true.

Definition (States of Affairs, Events, Situations)

- A set of infons that have the same location is called a state of affairs (soa).
- A set of infons with multiple locations is called an event (also, a course of affairs/events — coa).
- A situation is a collection of infons (a set or a proper class that is a non-well founded set).

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- Need of further refinement of the definitions of situations, states of affairs and courses of events, by considering:
    - Sets of infons may include inconsistency, e.g., by modelling contradictory or circular information.
    - The purpose is not to exclude the inconsistency, but to detect it (where possible).
    - There are definitions of (in)consistent situations.
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        - kinds of relations that are components of infons (there are classifications of verbs reflecting such differentiations)
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Example (A Situated Proposition)

$$(s \models \ll read, reader : x, readed : b, Loc : l_1; 1 \gg \wedge \quad (52a)$$
$$\ll book, arg : b, Loc : l_2; 1 \gg \wedge \quad (52b)$$
$$l_1 \circ l_2) \quad (52c)$$

- The proposition (52a)-(52c) is true iff
  - x reads b in the location l_1, in the situation s:

$$s \models \ll read, reader : x, readed : b, Loc : l_1; 1 \gg \quad (53)$$

  - b is having the property *book* in l_2, in the situation s:

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Quantificational scheme in Situation Semantics

Semantic quantifiers as relations between types of situated objects:

$$\Big(s \models \ll every,\ [x/(s_i \models \ll student, x, l_i; 1 \gg)],\ [y/(s_j \models \ll walk, y, l_j; 1 \gg)],\ l; 1 \gg \Big) \tag{56a}$$

$$\Big(s \models \ll some,\ [x/(s_i \models \ll student, x, l_i; 1 \gg)],\ [y/(s_j \models \ll walk, y, l_j; 1 \gg)],\ l; 1 \gg \Big) \tag{56b}$$

$$\Big(s \models \ll two,\ [x/(s_i \models \ll student, x, l_i; 1 \gg)],\ [y/(s_j \models \ll walk, y, l_j; 1 \gg)],\ l; 1 \gg \Big) \tag{56c}$$

- The proposition $pu(u, l, x, y, \alpha)$, where

$$pu(u, l, x, y, \alpha) \equiv (u \models \ll tells\_to, x, y, \alpha, l; 1 \gg) \quad (57)$$

$pu(u, l, x, y, \alpha)$ states that the situation u is an utterance situation.

- The proposition $pu(u, l, x, y, \alpha)$ is true iff u supports the uttering act:

$$u \models \ll tells\_to, x, y, \alpha, l; 1 \gg \quad (58)$$

i.e., iff
  - x is the speaker agent in u
  - y is the listener agent in u
  - l is the space-time location of the act of x uttering α
  - α is the expression uttered in u by the speaker agent x
- The type of an utterance situation is

$$ru(l, x, y, \alpha) \equiv [u \mid pu(u, l, x, y, \alpha)] \quad (59)$$

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$$rsp(u, l, y, \alpha) \equiv [x \mid pu(u, l, x, y, \alpha)] \tag{61}$$

- the type of a listener agent in u is:

$$rlst(u, l, x, \alpha) \equiv [y \mid pu(u, l, x, y, \alpha)] \tag{62}$$

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- in u, x is the speaker agent and y is the listener agent iff u supports an uttering act:

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Speaker's References: referent agents

- the type of the speaker's referent agent of the expression α

$$r_\alpha(u, l, x, y) = [z \mid q(u, l, x, y, z, \alpha)] \qquad (65)$$

where $q(u, l, x, y, z, \alpha)$ is a proposition such as (66a)

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$$(u^{ru(l,x,y,\alpha)} \models \qquad (66b)$$

$$\ll \textit{refers-to}, x^{rsp(u,l,y,\alpha)}, z, \alpha, l^{rdl(u,x,y,\alpha)}; 1 \gg) \qquad (66c)$$

The proposition $q(u, l, x, y, z, \alpha)$ in (66a) states that

- in the utterance $u^{ru(l,x,y,\alpha)}$, the speaker $x^{rsp(u,l,y,\alpha)}$ refers to the referent agent z of the expression α

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Speaker's denotations of referent agents

Denotation of a proper name, e.g., MARIA, as a referent agent

- a referent agent z^r: determined by a reference restriction r,
- in an utterance situation (context) u,
- by a speaker agent $x^{rsp(u,l,y,\alpha)}$

where the type restriction r may be

- general, sincere reference

$$r = [z \,|(u \models \ll refers\_to\_by, x^{rsp(u,l,y,\alpha)}, z, \text{MARIA}, l^{rdl}; 1 \gg) \land (u \models \ll named, \text{MARIA}, z; 1 \gg)]$$

- belief reference

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Linguistic meaning vs. interpretations with respect to different agents

- A restricted (constrained) utterance situation $u^{[u|pu(u,l,x,z,\alpha)]}$ is restricted by the proposition:

$$pu(u,l,x,y,\alpha) = (u \models \ll tells\_to, x, y, \alpha, l; 1 \gg) \qquad (67)$$

which introduces:

- pure linguistic meaning of α
- interpretation of the utterance of α with respect to various agents:
  - the speaker
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$$pu(u,l,x,y,\alpha) = (u \models \ll \mathit{tells\_to}, x, y, \alpha, l; 1 \gg) \qquad (67)$$

which introduces:

- pure linguistic meaning of α
- interpretation of the utterance of α with respect to various agents:
  - the speaker
  - various listeners
  - actual vs. intended or (mis)understood agents

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Definitions and Notations: pure linguistic meaning

Assume that

- $\sigma(x, l)$ is a parametric infon, with x and l among its parameters; l is a parameter for a space-time location, i.e.:

$$\sigma(x, l) : \text{INFON} \tag{68a}$$

$$\text{such that } x, l : \text{PAR}; \quad l : \text{LOC}; \quad x, l \text{ - free in } \sigma \tag{68b}$$

- $\sigma(x, l)$ is the linguistic information contributed by φ.
- T_s is the type of the situation described by the expression φ, in ("ignoring") abstraction of an utterance u of φ:

$$T_s = \lambda s[x \mid (s \models \sigma(x, l))] \tag{69a}$$

$$T_{s,l} = \lambda s, l[x \mid (s \models \sigma(x, l))] \tag{69b}$$

where s is a parameter for a situation described by a potential utterance u of the sentence φ.

Situational Types contributed by Parts of Speech in Sentences

- Verbs and Common Nouns, as components of sentences, designate types
- s, l are parameters for the *resource situation and location* of the components "book" / "read":

$$t_{book} = \lambda s, l[x \mid (s \models \ll book, x, l; 1 \gg)] \tag{70a}$$

$$t_{read} = \lambda s, l \lambda x[y \mid (s \models \ll read, y, x, l^{[l|l \circ l^{rdl}]}; 1 \gg)] \tag{70b}$$

Given that T is the type of a described situation and location

$$T = \lambda s, l[x \mid (s \models \sigma(x, l))] \tag{71}$$

- the speaker's reference c is a function over the parameters of T
- The extension $\mathcal{E}(T, c)$ of the type T, with respect to c, is the set of all objects $c'(x)$ of type T in the described situation $c(s)$ and location $c(l)$:

$$\mathcal{E}(T, c) = \{c'(x) \mid c(s) \models c'(\sigma(x))), \\ \text{where } c' \text{ differs from } c \text{ only possibly for } x\}.$$

- $\mathcal{E}(t_{book}, c)$ is the extension of the type of objects being a book in a resource situation $c(s_2)$, $c(l_2)$:

$$\mathcal{E}(t_{book}, c) = \{b \mid c(s_2) \models \ll book, b, c(l_2); 1 \gg\} \tag{72}$$

- $\mathcal{E}(t_{read}, c)$ is the extension of the type of objects a being read by an individual $c(y^r) = m$:

$$c(y^r) = m \tag{73}$$

$$\mathcal{E}(t_{read}, c) = \{a \mid c(s_1) \models \ll read, c(y^r), a, c(l_1^{[l|l \circ l^{rdl}]}); 1 \gg\} \tag{74}$$

where c is the function of the speaker's references in the utterance (context) u, e.g.,
$c(y^r)$ can be $c(y^r) = m$ to which the speaker refers to by the name MARIA

$$r = [z \mid (u \models \ll refers\_to\_by, x^{rsp(u,l,y,\alpha)}, z, \text{MARIA}, l^{rdl}; 1 \gg) \wedge (u \models \ll named, \text{MARIA}, , z; 1 \gg)]$$

- The type T of the situation s described by an utterance of (75), wrt resource situations s_1, s_2 and space-time locations l_1, l_2 is:

$$T \equiv [s, s_1, s_2, l_1, l_2 \mid (s \models \ll exist, [x \mid (s_2 \models \ll book, x, l_2; 1 \gg)], [x \mid (s_1 \models \ll read, y^r, x, l_1^{[l|l \circ l^{rdl}]}; 1 \gg)]; 1 \gg)]$$

- In a given utterance u, with speaker's reference c, s.t.
  - $c(s) = \dot{s}$ (the described situation)
  - $c(s_1) = \dot{s}_1$, $c(s_2) = \dot{s}_2$ (the resource situations)
  - $c(l_1) = \dot{l}_1$, $c(l_2) = \dot{l}_2$ (the resource locations)
- the proposition P expressed by the speaker is

$$P \equiv (\dot{s} \models \ll exist, [x \mid (\dot{s}_2 \models \ll book, x, \dot{l}_2; 1 \gg)], [x \mid (\dot{s}_1 \models \ll read, y^r, x, \dot{l}_1^{[l|l \circ l^{rdl}]}; 1 \gg)]; 1 \gg)$$

Applications that use the formal notions in this lecture

The notions introduced here are model-theoretic, i.e., they are per se semantic. They represent objects in mathematical structures of typed information, which has components including situations and space-time locations.

The following collection of papers uses such objects for computational semantics of human language:

- *Generalized Quantification in Situation Semantics.*
  See Loukanova [9]
- *Quantification and Intensionality in Situation Semantics.*
  See Loukanova [10]
- *Russellian and Strawsonian Definite Descriptions in Situation Semantics.*
  See Loukanova [8]

Some other applications that use Situation Theory

Situation Semantics for computational analysis of human language.

- Head-driven Phrase Structure Grammar (HPSG)
  See Pollard and Sag [12, 13]
- semantic analysis of questions
  See Ginzburg and Sag [7])
- semantics of tense and aspect, in settings of logic programing, from cognitive perspective
  See Lambalgen and Hamm [15]
- Minimal Recursion Semantics (MRS), for handling scope ambiguities (see Copestake et al. [4]

Existing and potential applications

- Type-theoretic syntax-semantics interfaces
  - programming languages
  - algorithm specifications: higher-order type theory of algorithms
  - data basis
  - information representation systems, e.g., in
    - health and medical systems
    - medical sciences
    - legal systems
- Syntax-semantics interface in grammar systems
- Applications to:
  - Human language processing
  - AI
  - Neuroscience

THANKS!

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